Economics Lecture 3

2016-17

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Course Outline

1 Consumer theory and its applications

1.1 Preferences and utility

1.2 Utility maximization and uncompensated demand

1.3 Expenditure minimization and compensated demand

1.4 Price changes and welfare

1.5 Labour supply, taxes and benefits

1.6 Saving and borrowing

2 Firms, costs and profit maximization

2.1 Firms and costs

2.2 Profit maximization and costs for a price taking firm

3. Industrial organization

3.1 Perfect competition and monopoly

3.2 Oligopoly and games

1.2 Utility maximization and uncompensated demand

1.2 Utility maximization and uncompensated demand

1. Budget line and budget set

2. Definition of uncompensated demand

3. Tangency and corner solutions

4. Finding uncompensated demand with Cobb-Douglas utility

5. The effects of changes in prices and income on uncompensated demand

6. Demand curves

7. Elasticity

8. Normal and inferior goods

9. Substitutes and complements

10. Finding uncompensated demand with perfect complements utility

11. Finding uncompensated demand with perfect substitutes utility

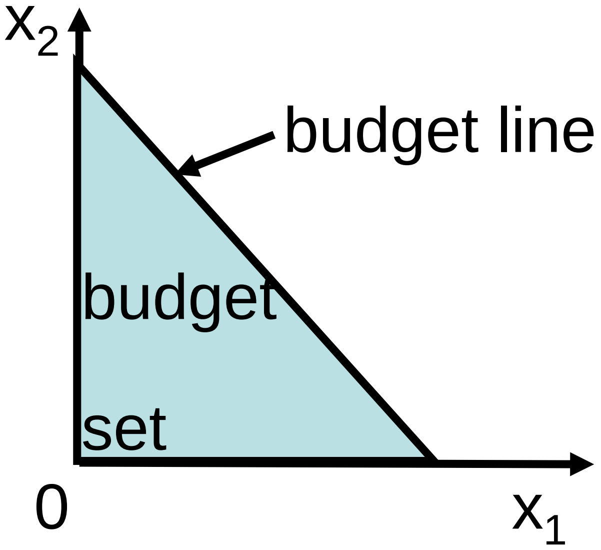
Utility maximization and uncompensated demand

1. Budget line and budget set

1. The budget set and budget line

Assume that it is impossible to consume negative quantities.

Notation		
quantities	x_1	x_2
prices	p_1	p_2
income	m	



Budget line $p_1x_1 + p_2x_2 = m$

Budget set points with $x_1 \geq 0$, $x_2 \geq 0$ $p_1x_1 + p_2x_2 \leq m$.

What is the gradient of the budget line?

budget line $p_1x_1 + p_2x_2 = m$

Rearranging gives $p_2x_2 = -p_1x_1 + m$

so $x_2 = -(p_1/p_2)\,x_1 + (m/p_2)$

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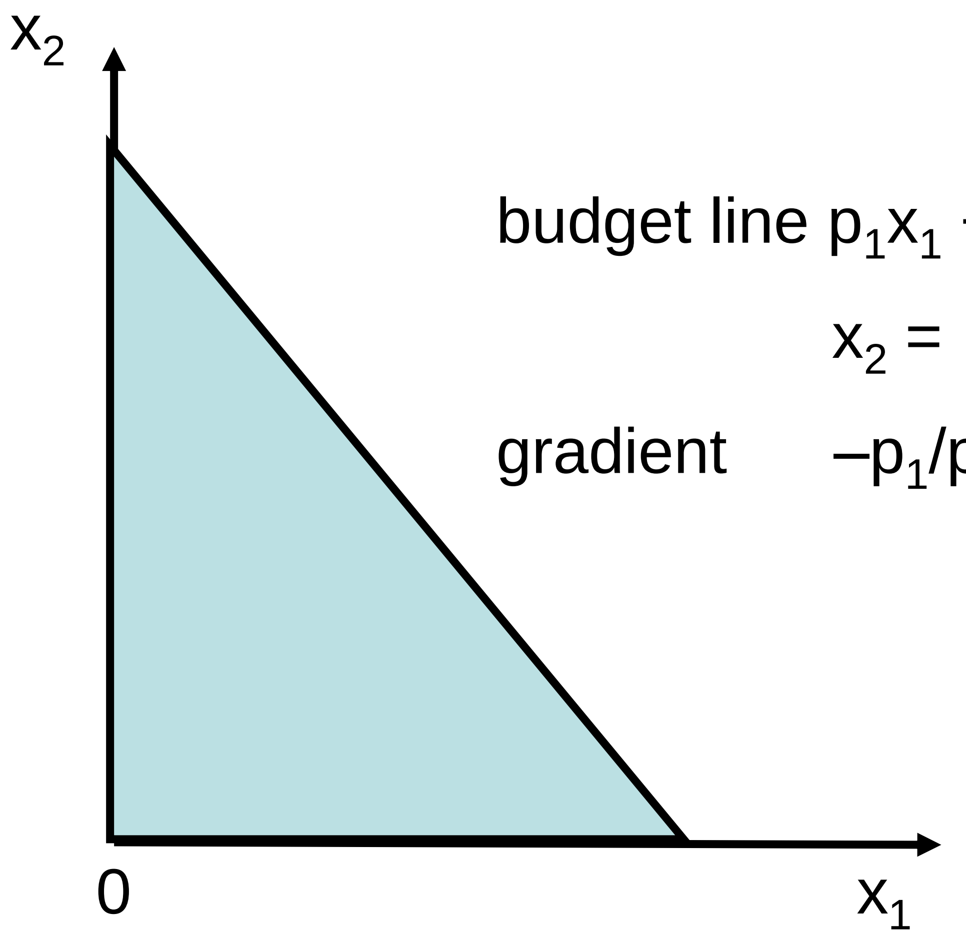
gradient

Where does the budget line meet the axes?

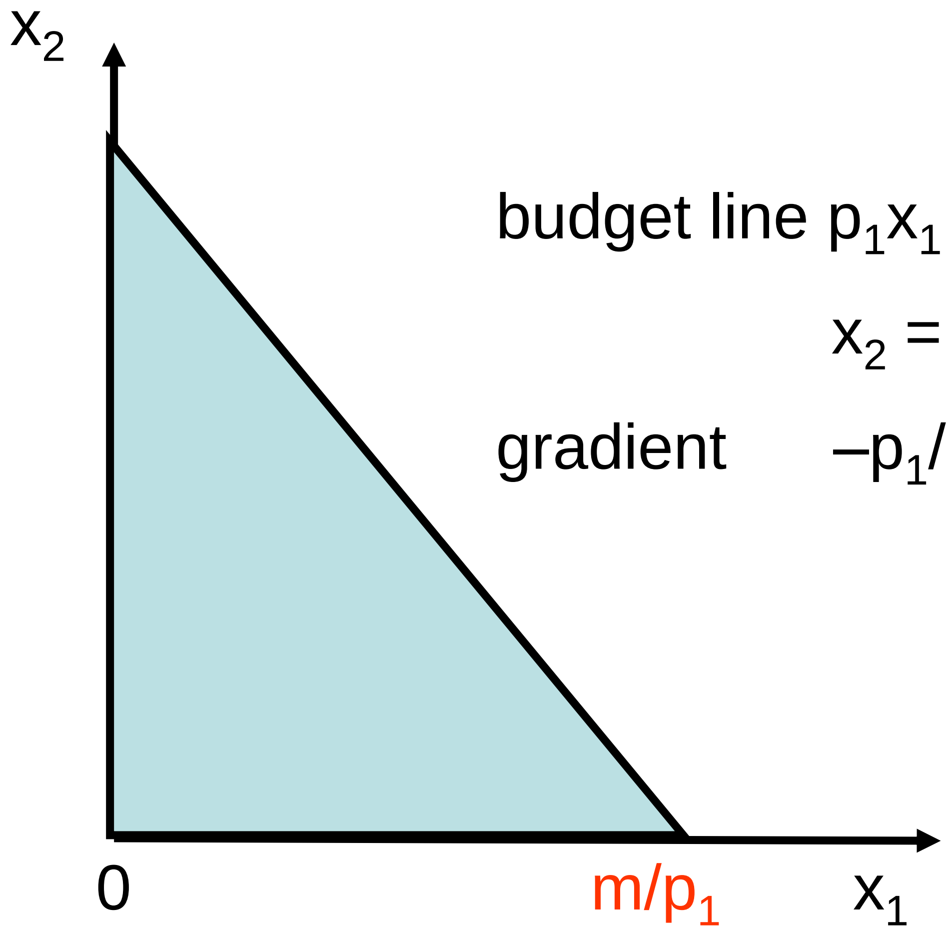


budget line $p_1x_1 + p_2x = m$

$x_2 = -(p_1/p_2)\ x_1 + (m/p_2)$

gradient $-p_1/p_2$.

Where does the budget line meet the axes?



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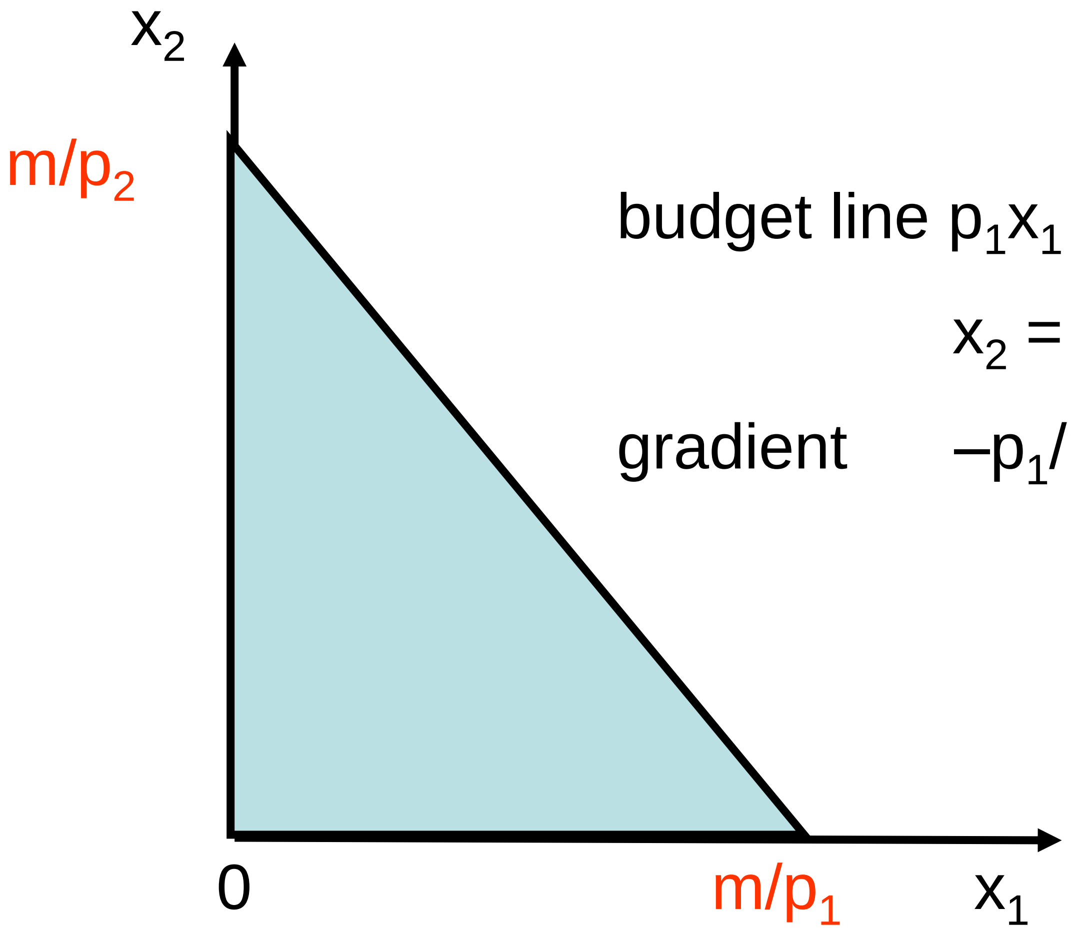

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gradient $-p_1/p_2$.

Utility maximization and uncompensated demand

2. Definition of uncompensated demand

2. Definition of uncompensated demand functions

Definition:

The consumer's ***demand functions***

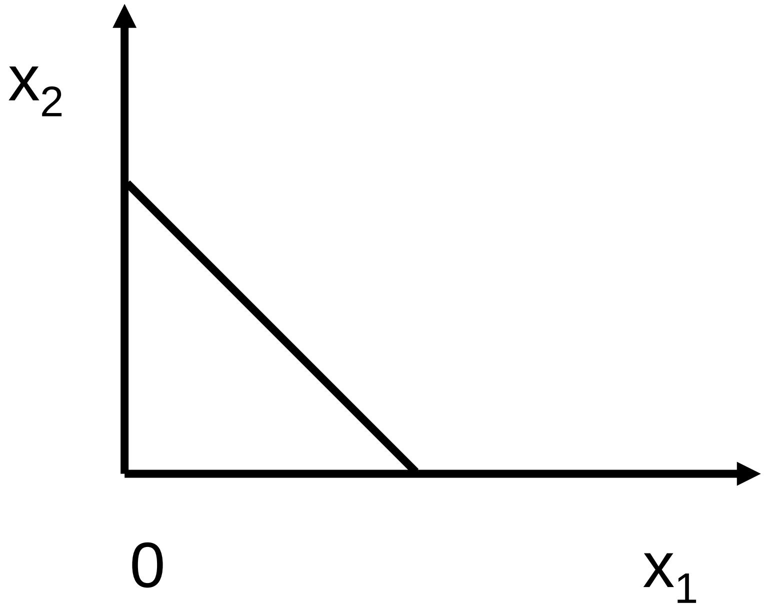
$x_1(p_1,p_2,m)$ and $x_2(p_1,p_2,m)$ maximize utility $u(x_1,x_2)$

subject to the budget constraint $p_1 x_1 + p_2 x_2 \leq m$

and non negativity constraints $x_1 \geq 0$ $x_2 \geq 0$.

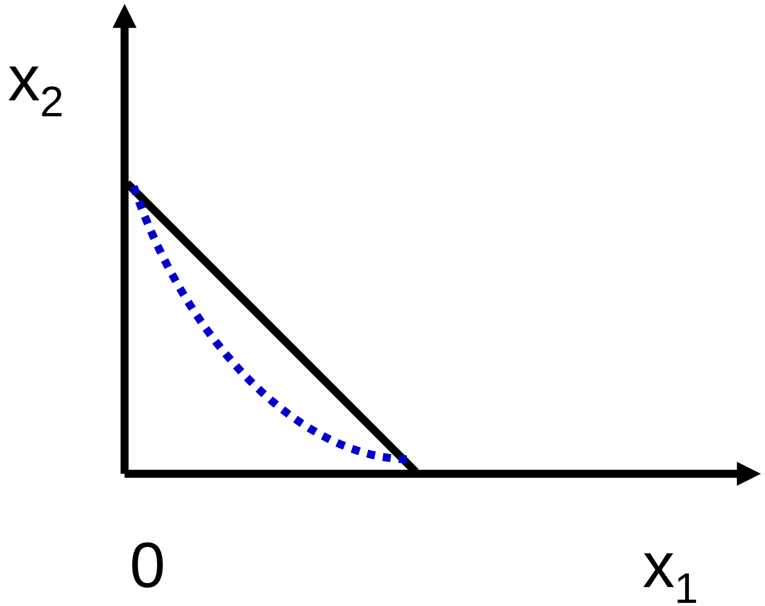
Later we call this "uncompensated demand".

Some books use the term "Marshallian demand".



To get <u>uncompensated</u> demand fix <u>income</u> and <u>prices</u> which fixes the budget line.

Get onto highest possible indifference curve.



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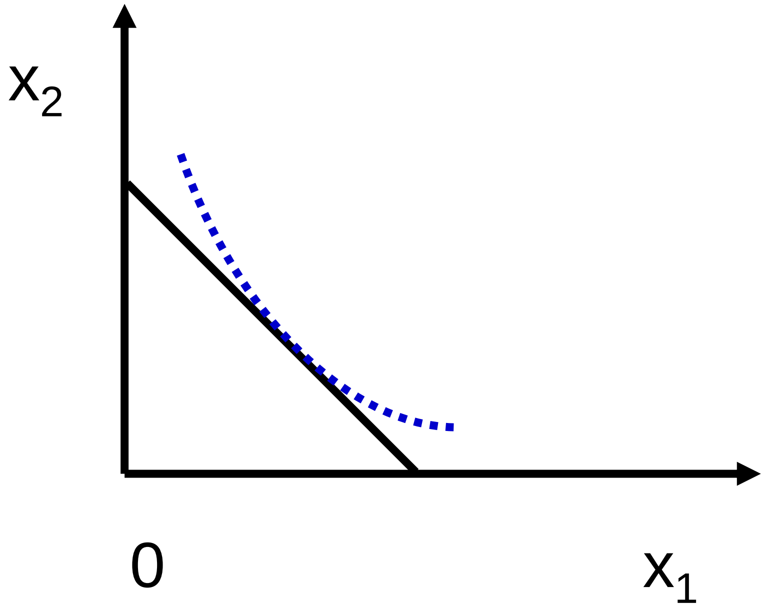

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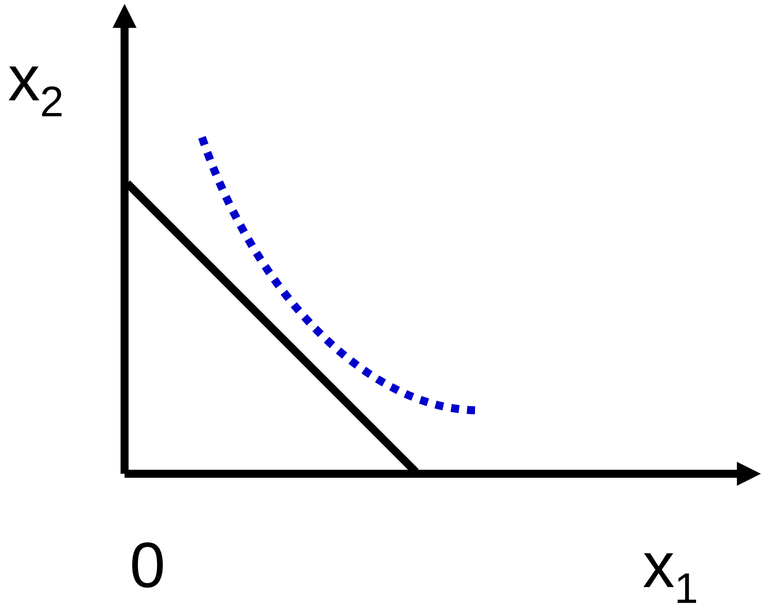

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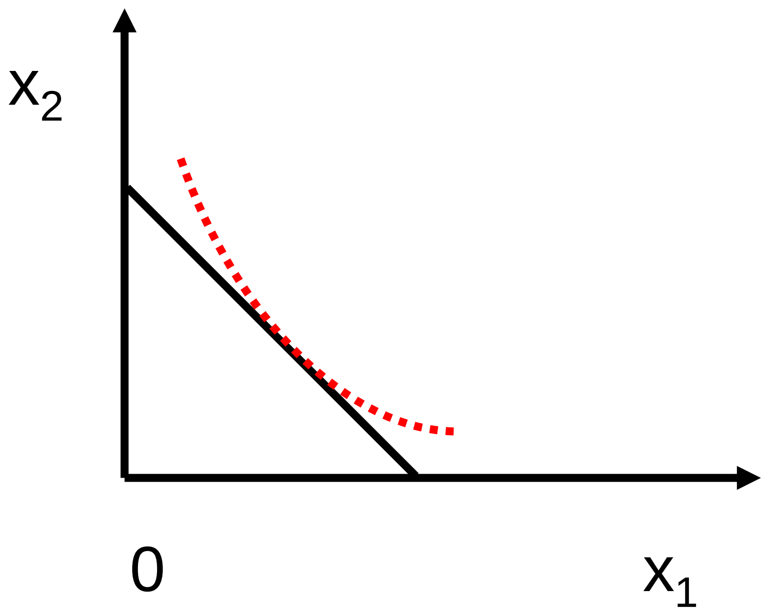


To get uncompensated demand fix income and prices which fixes the budget line.

Get onto highest possible indifference curve.

Compensated demand, Hicksian demand, is a demand function that holds utility fixed and minimizes expenditures. Uncompensated demand, Marshallian demand, is a demand function that maximizes utility given prices and wealth.

Examples of utility maximization (uncompensated demand)

Examples of utility maximization (uncompensated demand)

1. Cobb-Douglas utility
2. Perfect complements
3. Perfect substitutes

Examples of utility maximization (uncompensated demand)

For each example we will look at

- Indifference curve diagram
- Effect of prices on demand, own price and cross price elasticities
- Effect of income on demand, normal and inferior goods, income elasticity
- Demand curve diagram

8 steps for finding uncompensated demand

8 steps for finding uncompensated demand with differentiable utility

1. Write down the problem you are solving

2. What is the solution a function of?

3. Check for nonsatiation and convexity using calculus **if the utility function has partial derivatives**

   Explain their implications.

4. Use the tangency and budget line conditions.

8 steps for finding uncompensated demand

5. Draw a diagram based on the tangency and budget line conditions.

6. Remind yourself what you are finding and what it depends on.

7. Write down the equations to be solved.

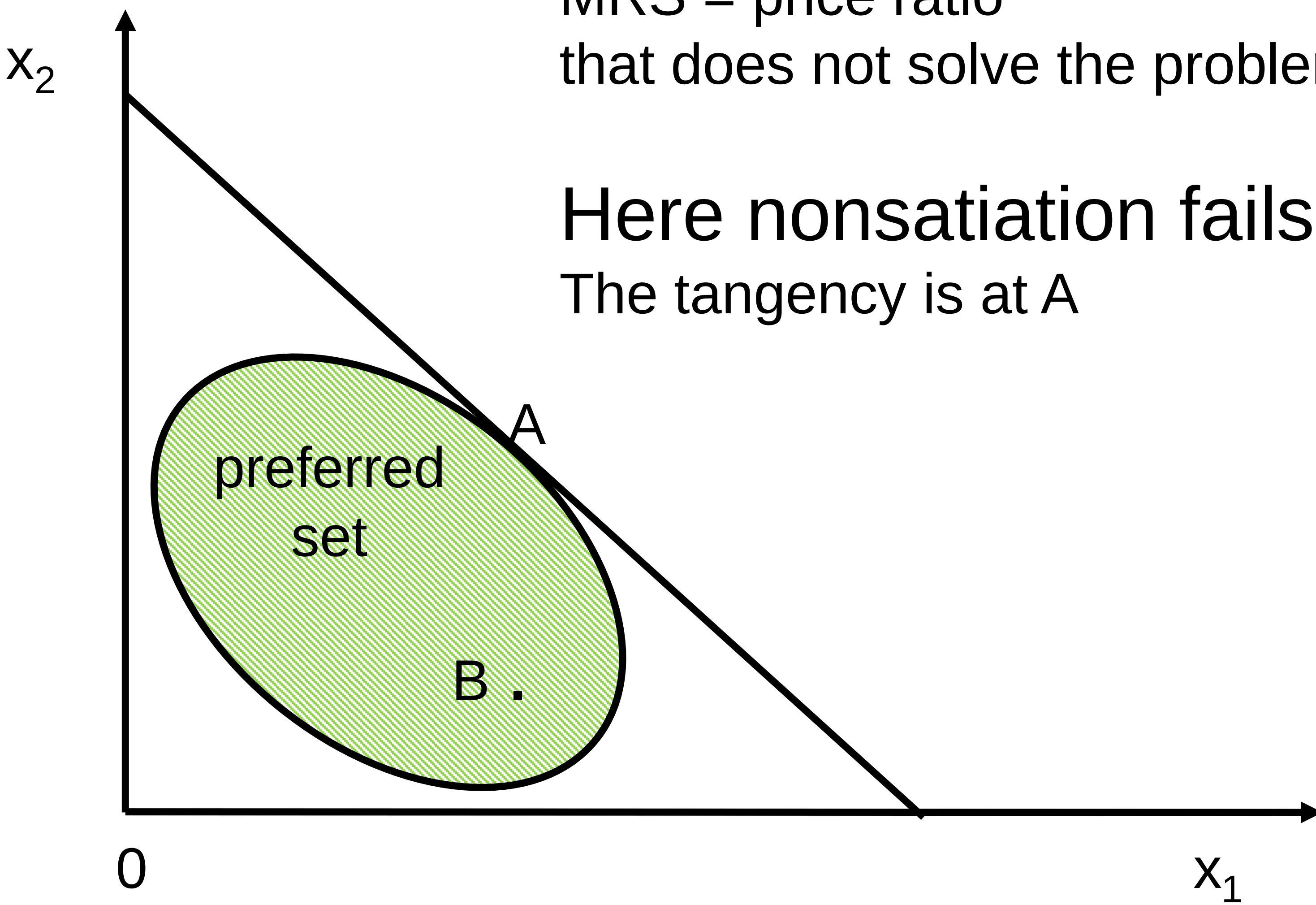
8. Solve the equations and write down the solution as a function. If at this point $x_1 \geq 0$ and $x_2 \geq 0$ you have found the utility maximizing point.

Why check for nonsatiation and convexity?

If they are not satisfied there can be a tangency point A where MRS = price ratio that does not solve the problem.

Here nonsatiation fails.

The tangency is at A



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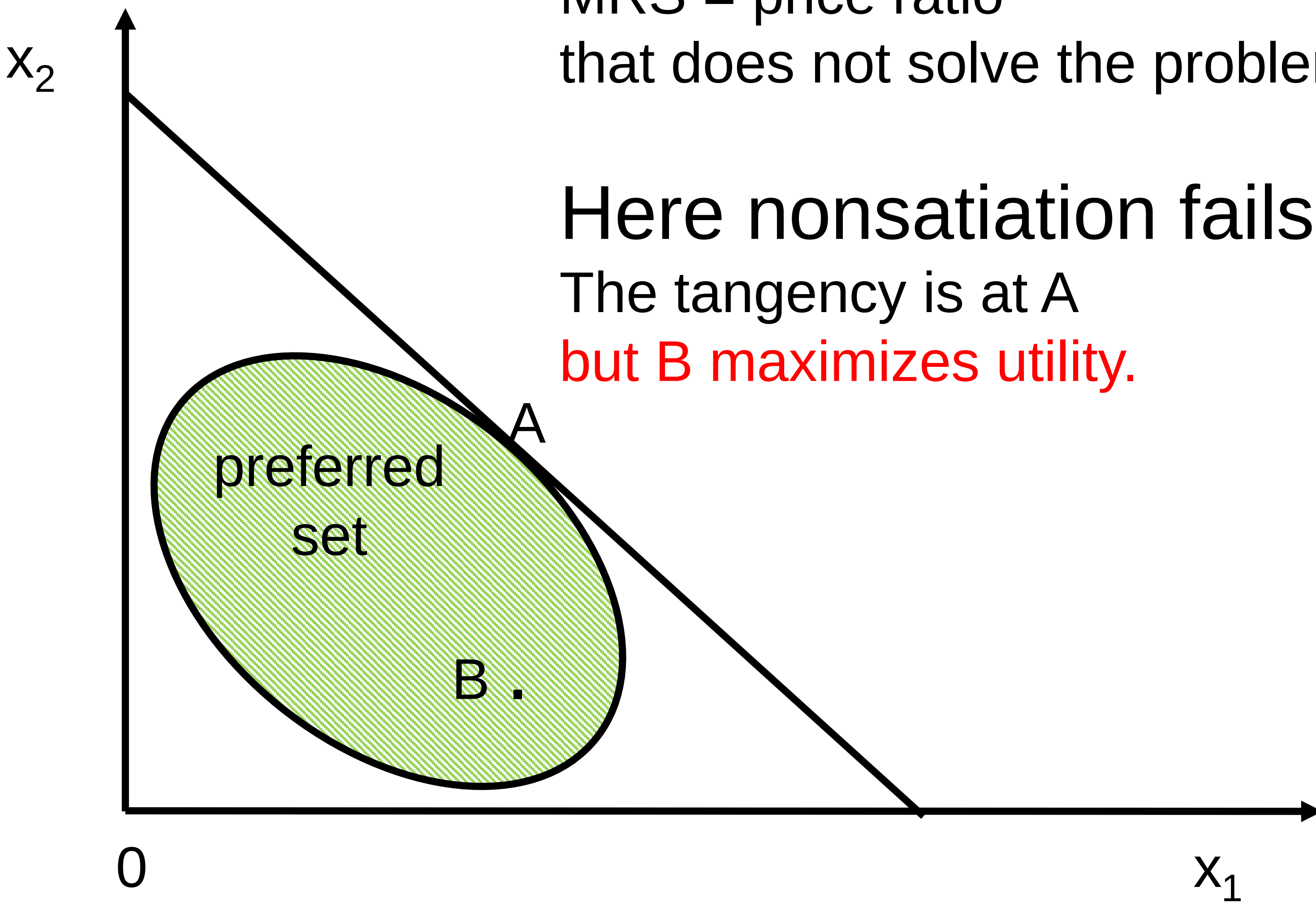


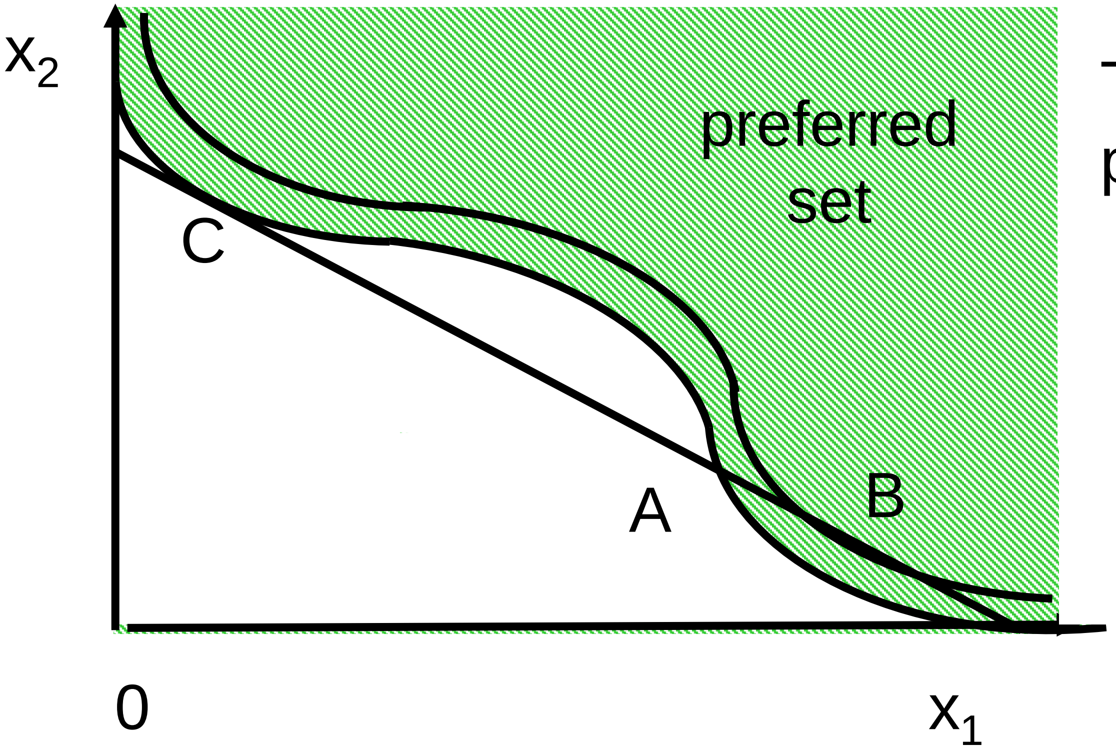
Here nonsatiation fails.

The tangency is at A
but B maximizes utility.

Why check for nonsatiation and convexity?

A point like A that is not a tangency

The point B with $x_1 > 0$ and $x_2 > 0$



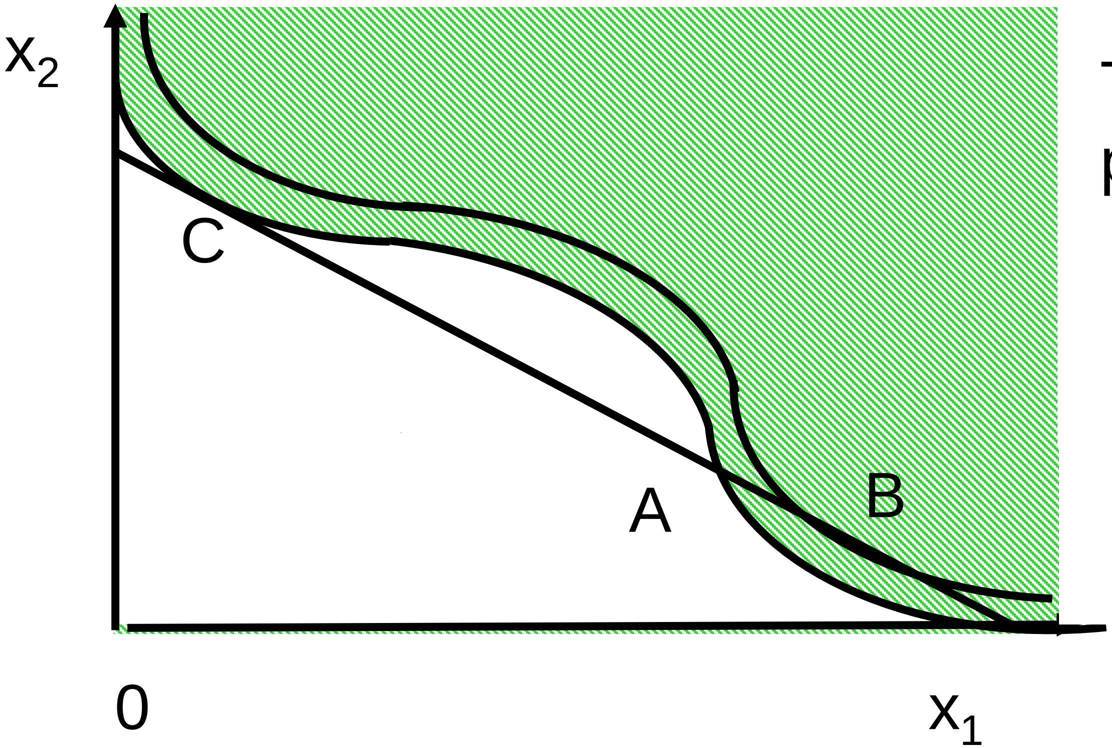
The point C is a tangency point but does

Here convexity fails

Why check for nonsatiation and convexity?

A point like A that is not a tangency cannot maximize utility.

The point B with $x_1 > 0$ and $x_2 > 0$



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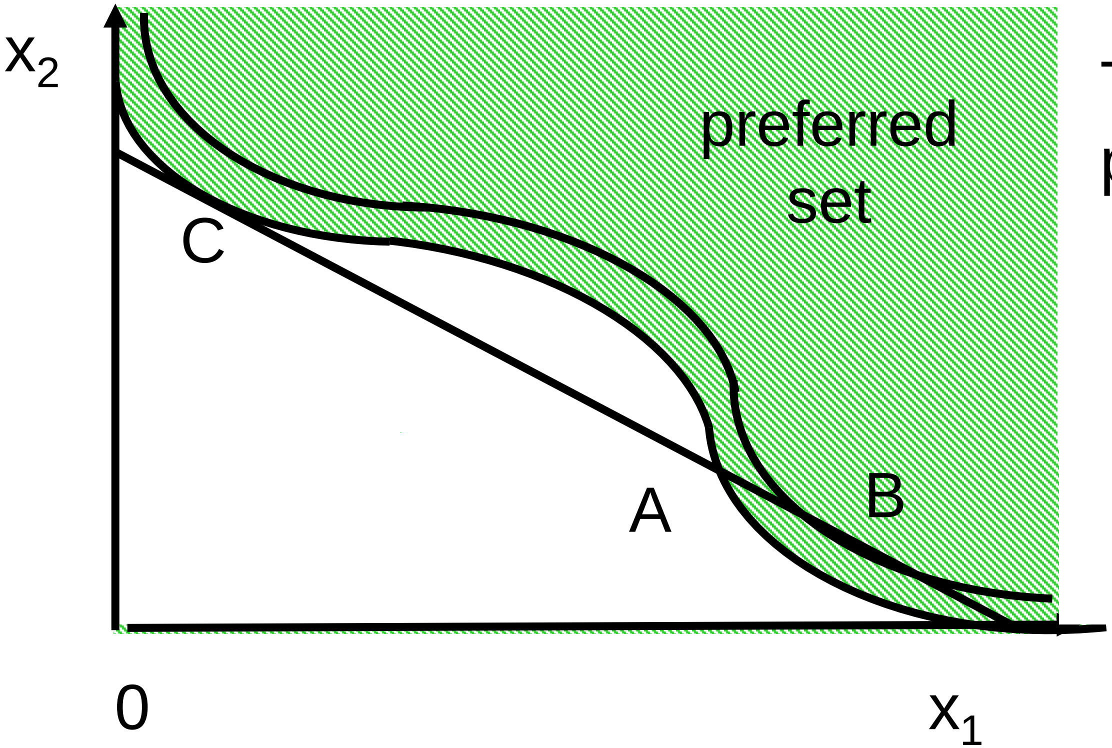

The point C is a tangency point but does

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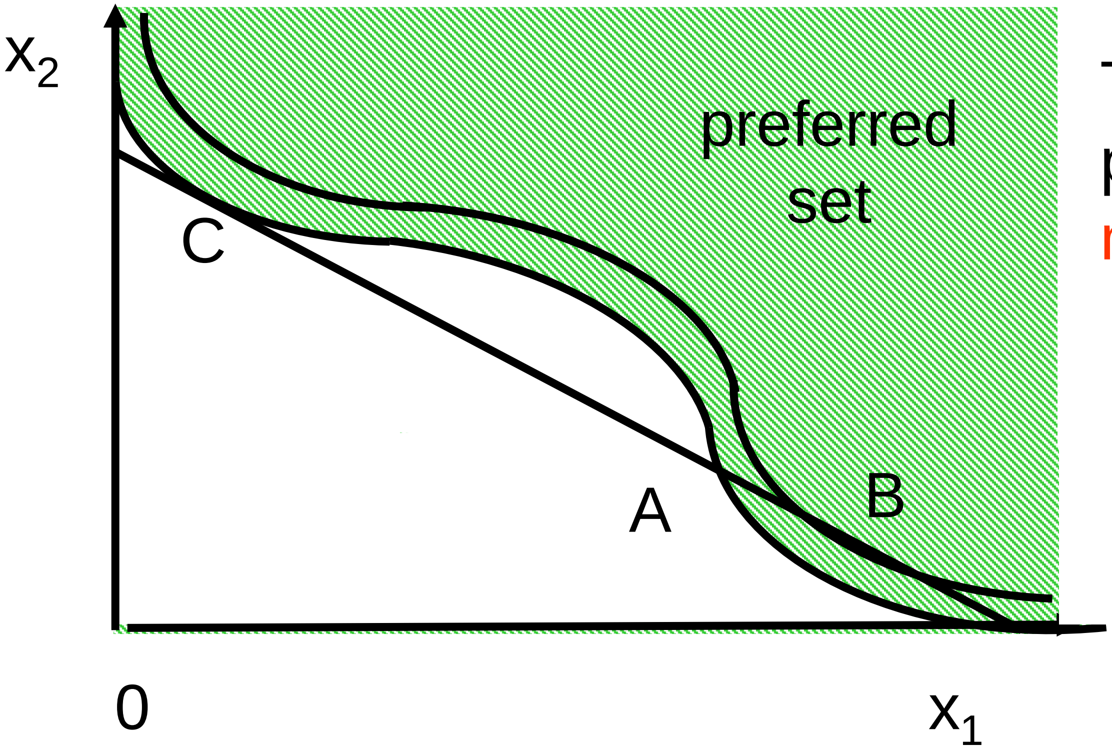
Why check for nonsatiation and convexity?

A point like A that is not a tangency cannot maximize utility.

The point B with $x_1 > 0$ and $x_2 > 0$ maximises utility.

It must be a tangency point.



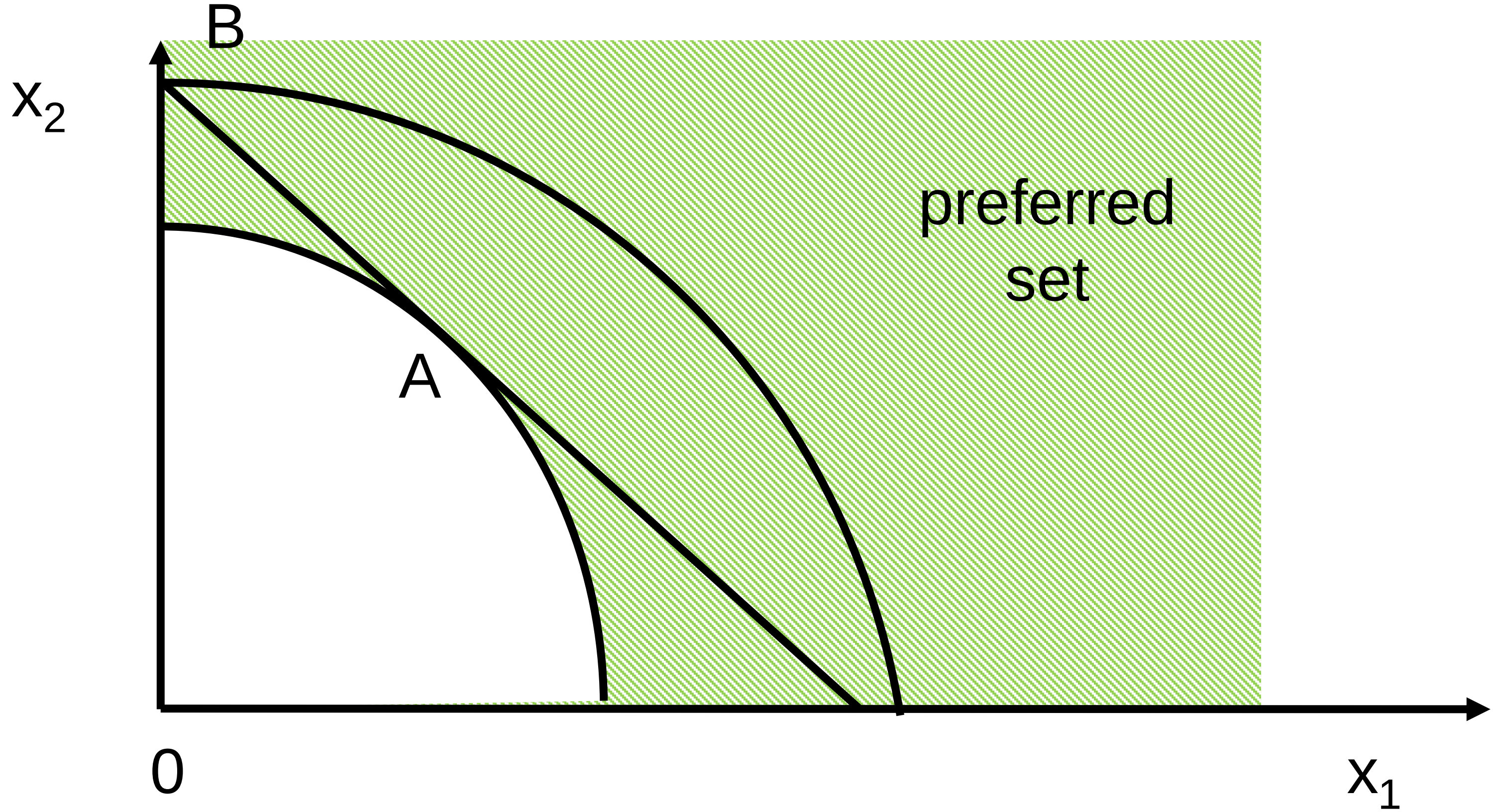
The point C is a tangency point but does not maximize utility.

Here convexity fails

Why check for nonsatiation and convexity?

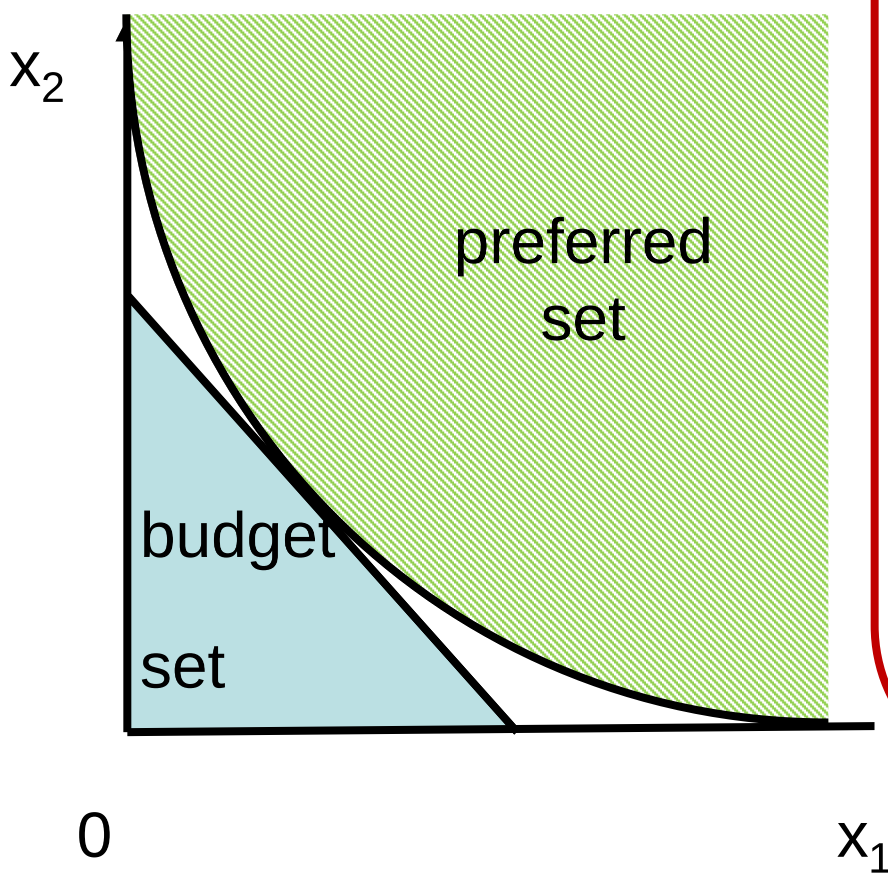
Here convexity fails.

The tangency is A



Logic of first order conditions

Very important.



If the nonsatiation and convexity conditions are satisfied then any tangency point at which

MRS = price ratio

$x_1 \geq 0, \quad x_2 \geq 0$

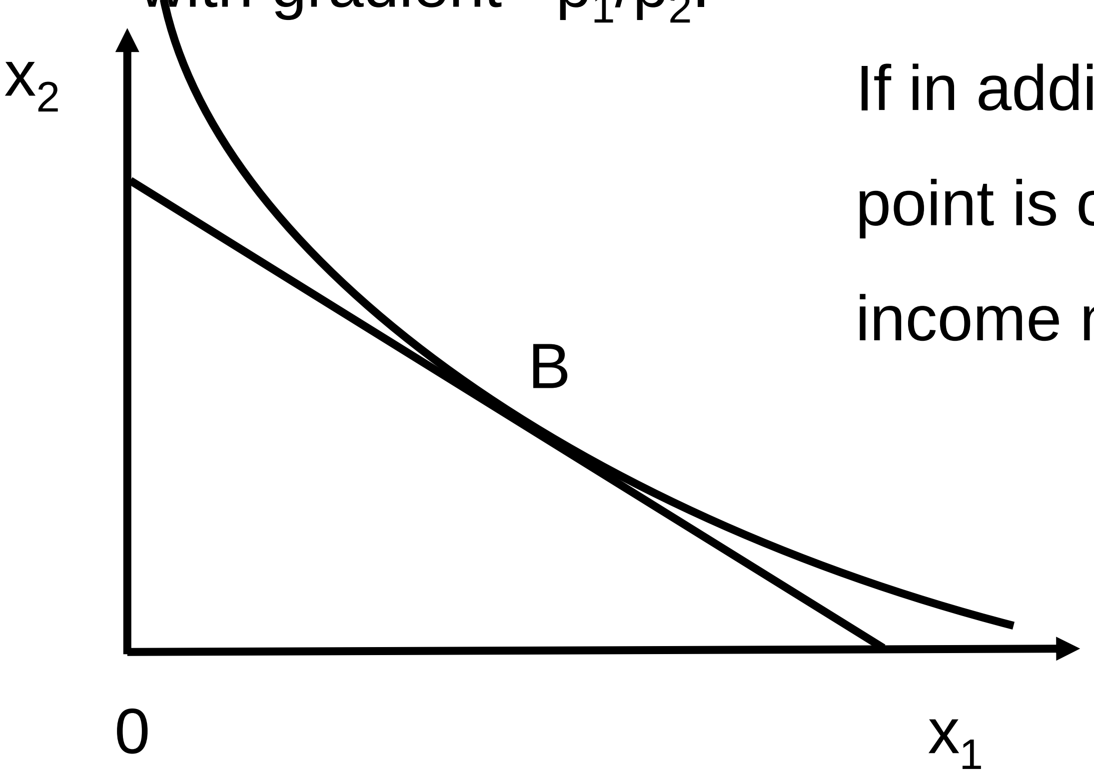
solves the utility maximizing problem.

Finding a tangency solution

The gradient of the indifference curve is – MRS.

The gradient of the budget line is – p_1/p_2.

If MRS = p_1/p_2 the point is tangent to some budget line with gradient $-p_1/p_2$.



If in addition $p_1x_1 + p_2x_2 = m$ the point is on the budget line with income m.

We have already found that

$$\text{MRS} = \frac{\dfrac{\partial u}{\partial x_1}}{\dfrac{\partial u}{\partial x_2}}$$

so MRS = price ratio requires that

$$\frac{\dfrac{\partial u}{\partial x_1}}{\dfrac{\partial u}{\partial x_2}} = \frac{p_1}{p_2}$$

Another way to look at the tangency condition

- Get Δx_1 more units of x_1

  increase in utility $\Delta x_1 \dfrac{\partial u}{\partial x_1}$

- Spend €1 more on x_1 gives $1/p_1$ more units of good 1

  - increase in utility $\dfrac{1}{p_1}\dfrac{\partial u}{\partial x_1}$

- Spend €1 less on x_2

  - fall in utility $\dfrac{1}{p_2}\dfrac{\partial u}{\partial x_2}$

- Spending €1 more on x_1 and €1 less on good 2

  increases utility if $$\frac{1}{p_1}\frac{\partial u}{\partial x_1} > \frac{1}{p_2}\frac{\partial u}{\partial x_2}$$

- Spending €1 less on good 1 and €1 more on good 2

  increases utility if $$\frac{1}{p_2}\frac{\partial u}{\partial x_2} > \frac{1}{p_1}\frac{\partial u}{\partial x_1}$$

- Utility maximization requires

$$\frac{1}{p_1}\frac{\partial u}{\partial x_1} = \frac{1}{p_2}\frac{\partial u}{\partial x_2}$$

- Utility maximization requires $\dfrac{1}{p_1}\dfrac{\partial u}{\partial x_1} = \dfrac{1}{p_2}\dfrac{\partial u}{\partial x_2}$

or $$\frac{\dfrac{\partial u}{\partial x_1}}{\dfrac{\partial u}{\partial x_2}} = \frac{p_1}{p_2}$$

i.e. MRS = price ratio

Finding uncompensated demand with Cobb-Douglas utility

$$u(x_1,x_2) = x_1^{2/5}x_2^{3/5}$$

4. Finding uncompensated demand with Cobb-Douglas utility

Step 1: What problem are you solving?

The problem is maximizing utility $u(x_1,x_2) = x_1^{2/5}x_2^{3/5}$

subject to non-negativity constraints $x_1 \geq 0$ $x_2 \geq 0$

and the budget constraint $p_1x_1 + p_2x_2 \leq m$.

Step 2: What is the solution a function of?

Demand is a function of prices and income so is

$x_1(p_1,p_2,m)$ $x_2(p_1,p_2,m)$

Finding uncompensated demand with Cobb-Douglas utility

Step 3: Check for nonsatiation and convexity

We have already done this, both are satisfied.

Easy to lose exam marks

Failing to say that

Because nonsatiation and convexity are satisfied any point on the budget line at which

MRS = price ratio, $x_1 \geq 0$ and $x_2 \geq 0$

solves the utility maximizing problem.

Finding uncompensated demand with Cobb-Douglas utility

Step 4: Use the tangency and budget line conditions

Because convexity and nonsatiation are satisfied any point with

$p_1x_1 + p_2x_2 = m$ so it is on the budget line

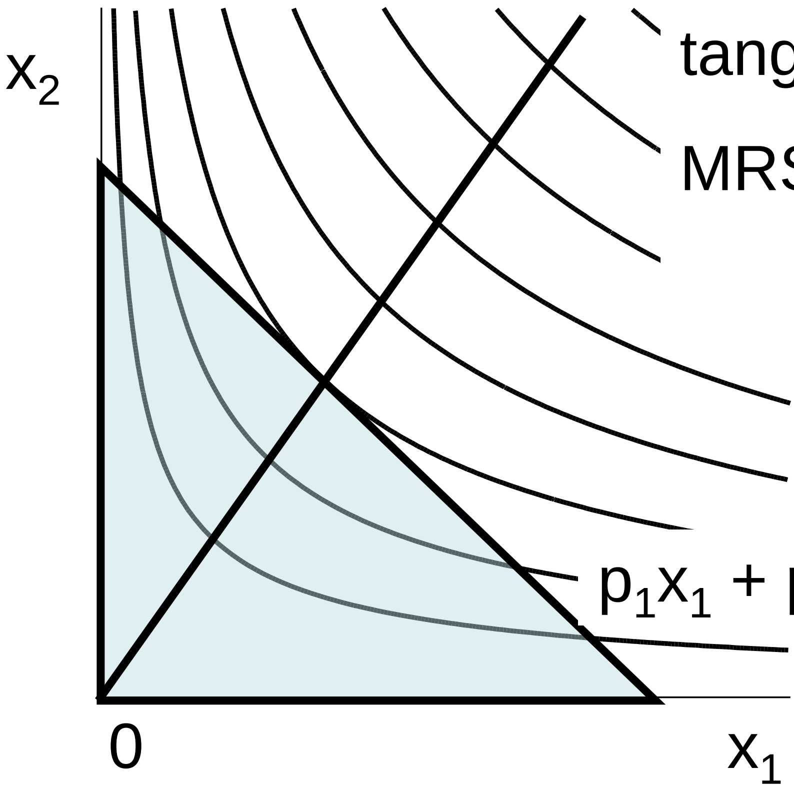
and MRS = $\frac{p_1}{p_2}$ solves the problem

here we have already found

$$MRS = -\frac{\frac{\partial u}{\partial x_1}}{\frac{\partial u}{\partial x_2}} = -\frac{\frac{2}{5}x_1^{-3/5}x_2^{3/5}}{\frac{3}{5}x_1^{2/5}x_2^{-2/5}} = -\frac{2x_2}{3x_1}$$

Finding uncompensated demand with Cobb-Douglas utility

Step 5: Draw a diagram based on the tangency and budget line conditions



Finding uncompensated demand with Cobb-Douglas utility

Step 6: Remind yourself what you are finding and what it depends on.

You are finding demand x_1 and x_2 which is a function of p_1, p_2 and m.

Step 7: Write down the equations to be solved.

The equations are $p_1x_1 + p_2x_2 = m$ and

$$\frac{2x_2}{3x_1} = \frac{p_1}{p_2}$$

Finding uncompensated demand with Cobb-Douglas utility

Step 8 solve the equations and write down the solution as a function.

(You will do some algebra here.)

Solving the equations simultaneously for x_1 and x_2 gives

(uncompensated) demand which is a function of p_1, p_2, m.

$x_1(p_1,p_2,m) = \frac{2}{5}\frac{m}{p_1}$ $\qquad$ $x_2(p_1,p_2,m) = \frac{3}{5}\frac{m}{p_2}$

because the conditions $x_1 \geq 0$, $x_2 \geq 0$ are satisfied.

An alternative approach: using Lagrangians

You can also use Lagrangians to find demand.

With two goods, the Lagrangian is not essential. You can base your analysis on graphs and simple algebra.

With more than 2 goods you have to use Lagrangians.

Homogeneity of uncompensated demand

5. The effects of changes in prices and income on uncompensated demand

- If all prices and income are multiplied by a number $k > 0$

  what happens?

If all prices and income are all multiplied by 2 what happens?

1. Demand for good 1 increases.
2. Demand for good 1 decreases.
3. ✓ Demand for good 1 does not change.
4. Demand for good 2 increases.
5. Demand for good 2 decreases.
6. ✓ Demand for good 2 does not change.

Mathematical definition of homogeneous functions

A function $f(z_1,z_2,z_3\ldots..z_n)$ is homogeneous of degree 0 if for all numbers $k > 0$

$$f(kz_1,kz_2,kz_3\ldots..kz_n) = k^0 f(z_1,z_2,z_3\ldots..z_n) = f(z_1,z_2,z_3\ldots..z_n).$$

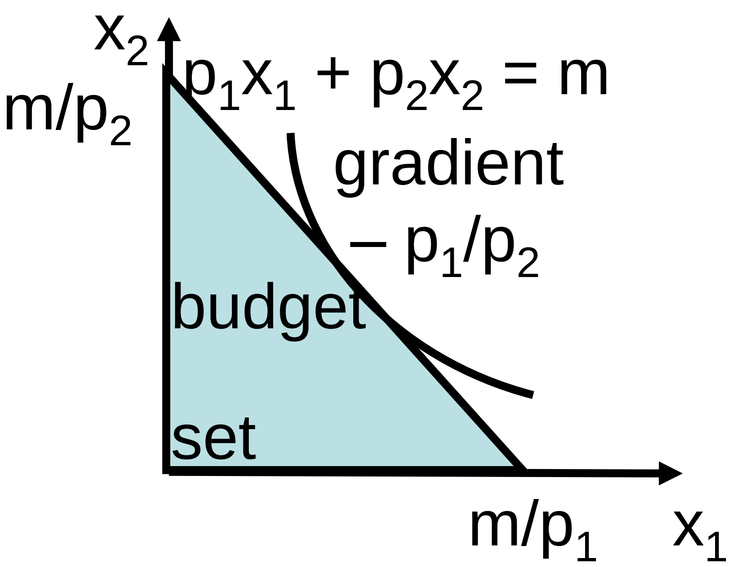
Multiplying $z_1,z_2,\ldots..z_n$ by $k > 0$ does not change the value of f.

A function $f(z_1,z_2,z_3\ldots..z_n)$ is homogeneous of degree one if for all numbers $k > 0$

$$f(kz_1,kz_2,kz_3\ldots..kz_n) = k^1 f(z_1,z_2,z_3\ldots..z_n) = kf(z_1,z_2,z_3\ldots..z_n)$$

Multiplying z_1, z_2 …z_n multiplies the value of f by k.

All prices and income are multiplied by k > 0.



How does the budget line change?

How does demand change?

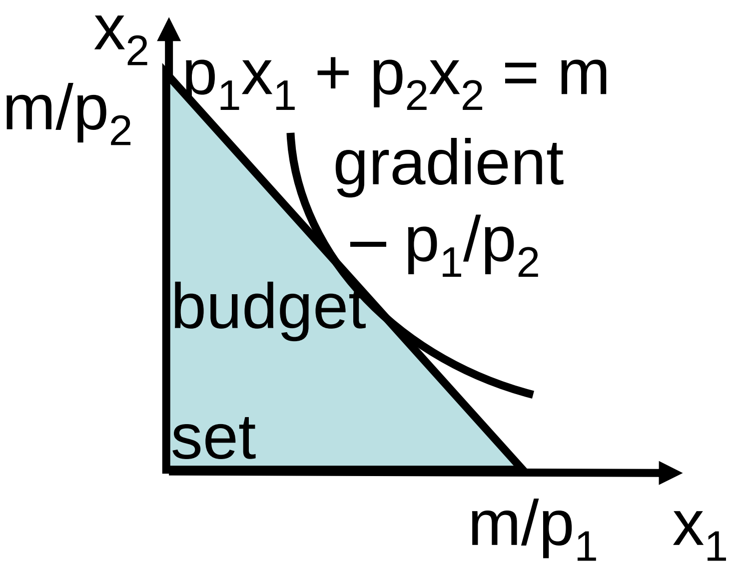
The consumer's demand functions $x_1(p_1,p_2,m)$ and $x_2(p_1,p_2,m)$ are in prices and income.

That is if $k > 0$

$x_1(kp_1,kp_2,km) =$

$x_2(kp_1,kp_2,km) =$

All prices and income are multiplied by k > 0.



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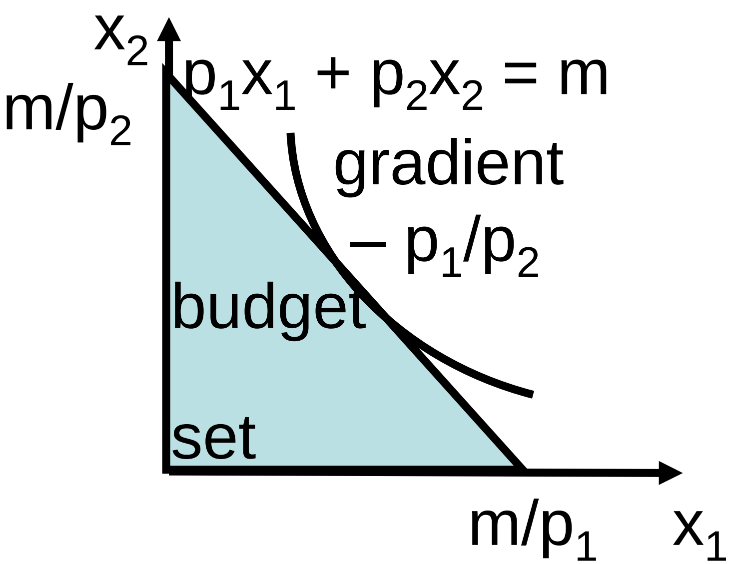

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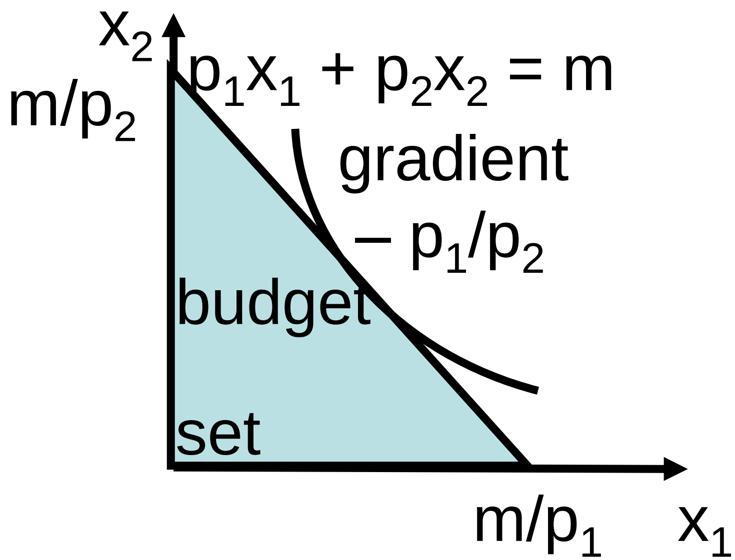
All prices and income are multiplied by k > 0.



How does the budget line change?

no change

How does demand change?

no change

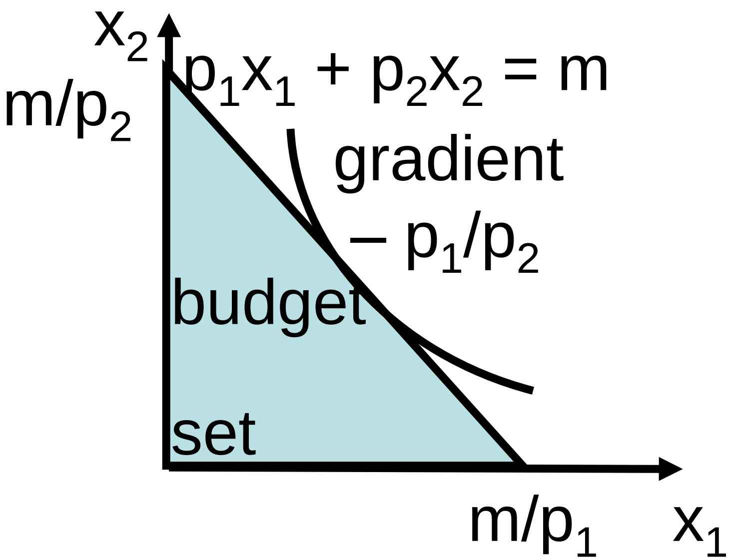
The consumer's demand functions $x_1(p_1,p_2,m)$ and $x_2(p_1,p_2,m)$ are **homogeneous of degree 0** in prices and income.

That is if $k > 0$

$x_1(kp_1,kp_2,km) =$

$x_2(kp_1,kp_2,km) =$

All prices and income are multiplied by k > 0.



How does the budget line change?

no change

How does demand change?

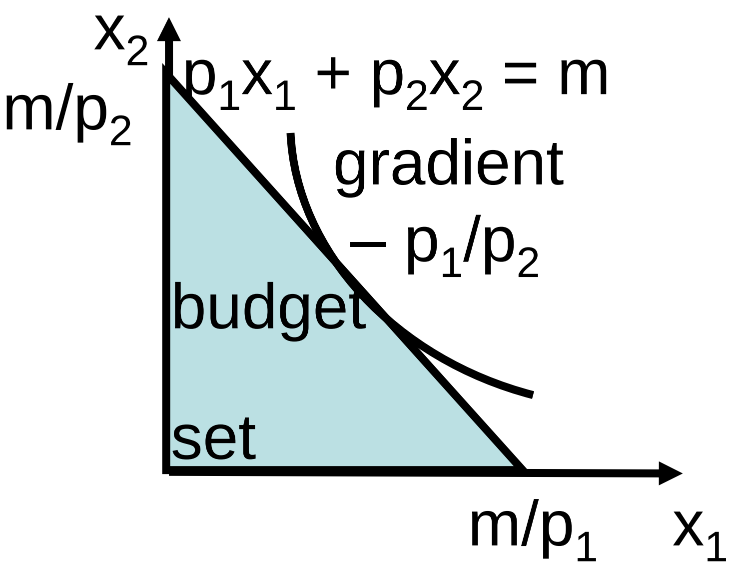
no change

The consumer's demand functions $x_1(p_1,p_2,m)$ and $x_2(p_1,p_2,m)$ are **homogeneous of degree 0** in prices and income.

That is if $k > 0$

$x_1(kp_1,kp_2,km) = x_1(p_1,p_2,m)$ $\qquad$ $x_2(kp_1,kp_2,km) =$

All prices and income are multiplied by k > 0.



How does the budget line change?

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How does demand change?

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The consumer's demand functions $x_1(p_1,p_2,m)$ and $x_2(p_1,p_2,m)$ are **homogeneous of degree 0** in prices and income.

That is if $k > 0$

$x_1(kp_1,kp_2,km) = x_1(p_1,p_2,m)$ $\qquad$ $x_2(kp_1,kp_2,km) = x_2(p_1,p_2,m)$

All uncompensated demand functions are homogeneous of degree 0 in prices and income

This means that if all prices and income are all multiplied by $k > 0$ demand does not change.

With Cobb-Douglas utility $u(x_1,x_2) = x_1^{2/5}x_2^{3/5}$

$$x_1(p_1,p_2,m) = \frac{2}{5}\frac{m}{p_1} \qquad x_2(p_1,p_2,m) = \frac{3}{5}\frac{m}{p_2}$$

The values of these functions do not change when p_1, p_2 and m are all multiplied by $k > 0$.

Easy to lose exam marks

Explain what happens to uncompensated demand when prices and income are all multiplied by 2

Saying nothing happens <u>because</u> uncompensated demand is homogeneous of degree 0 in prices

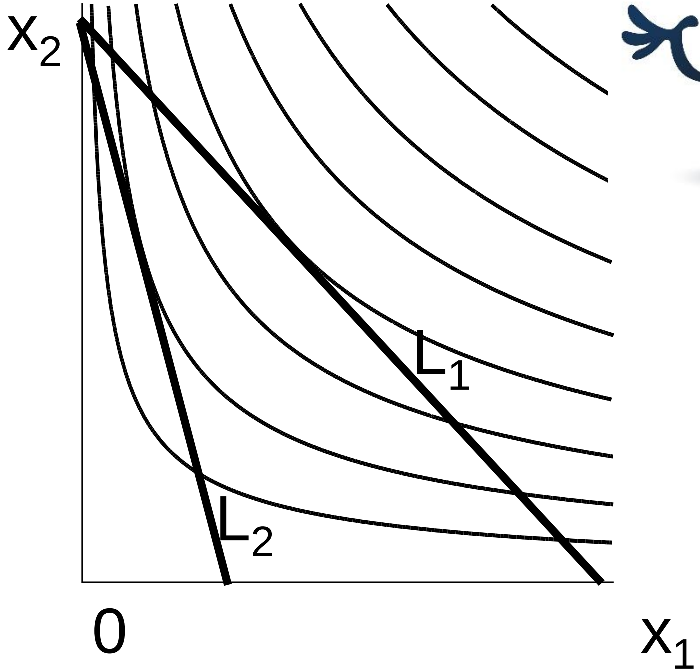
This is a statement of what happens – it is not an explanation.

Changes in demand and demand curves

TP

The budget line moves from L_1 to L_2. Is this due to

1. An increase in p_1
2. A decrease in p_1
3. An increase in p_2
4. A decrease in p_2
5. An increase in m
6. A decrease in m



p_1 increases, p_2 and m do not change. What happens to demand for goods 1 and 2?

With Cobb-Douglas utility $u(x_1,x_2) = x_1^{2/5}x_2^{3/5}$

$x_1(p_1,p_2,m) = \frac{2}{5}\frac{m}{p_1}$

$x_2(p_1,p_2,m) = \frac{3}{5}\frac{m}{p_2}$

p_1 increases, p_2 and m do not change. What happens to demand for goods 1 and 2?

1. Demand for good 1 increases.
2. ✓ Demand for good 1 decreases.
3. Demand for good 1 does not change.
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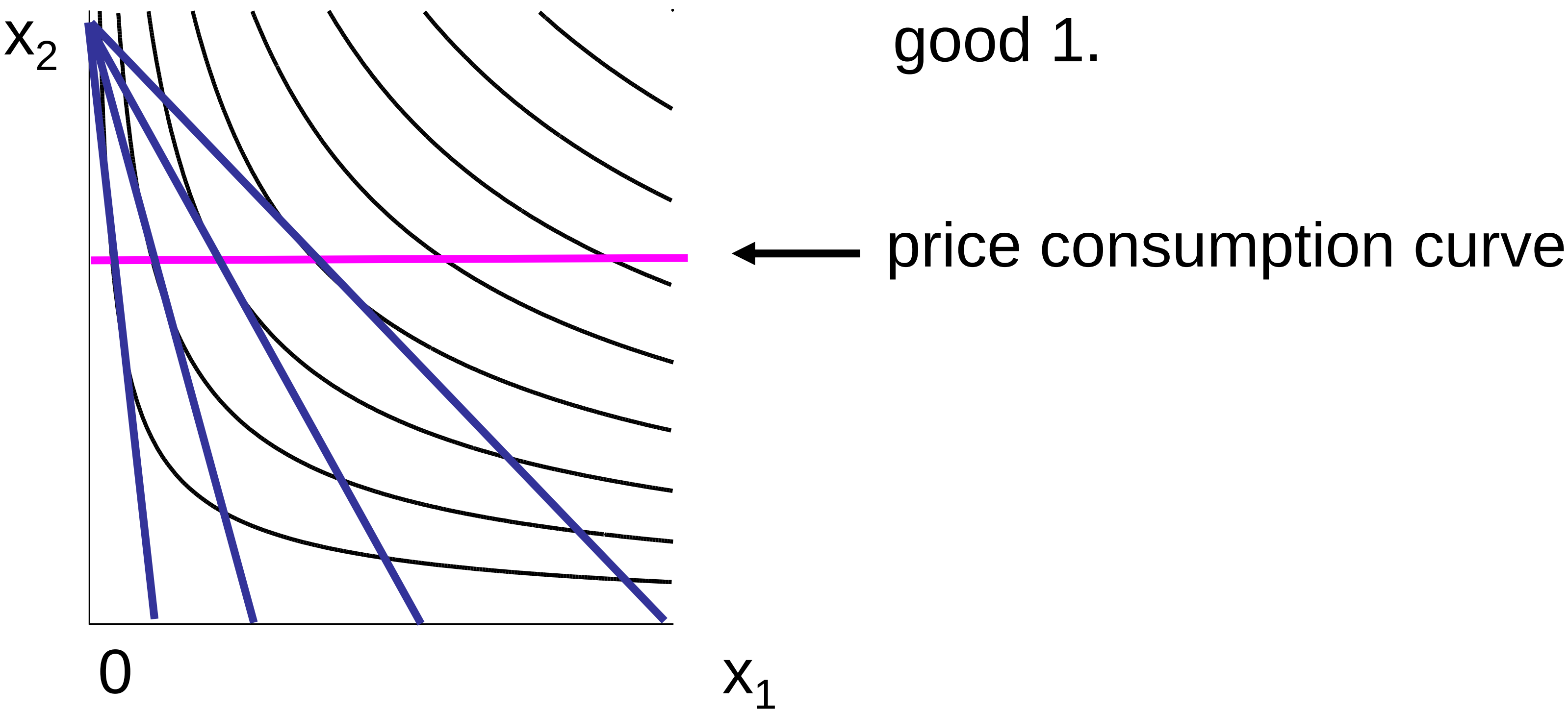
With Cobb-Douglas utility $u(x_1,x_2) = x_1^{2/5}x_2^{3/5}$

$$x_1(p_1,p_2,m) = \frac{2}{5}\frac{m}{p_1} \qquad x_2(p_1,p_2,m) = \frac{3}{5}\frac{m}{p_2}$$



Demand for good 2 is not affected by the price of good 1.

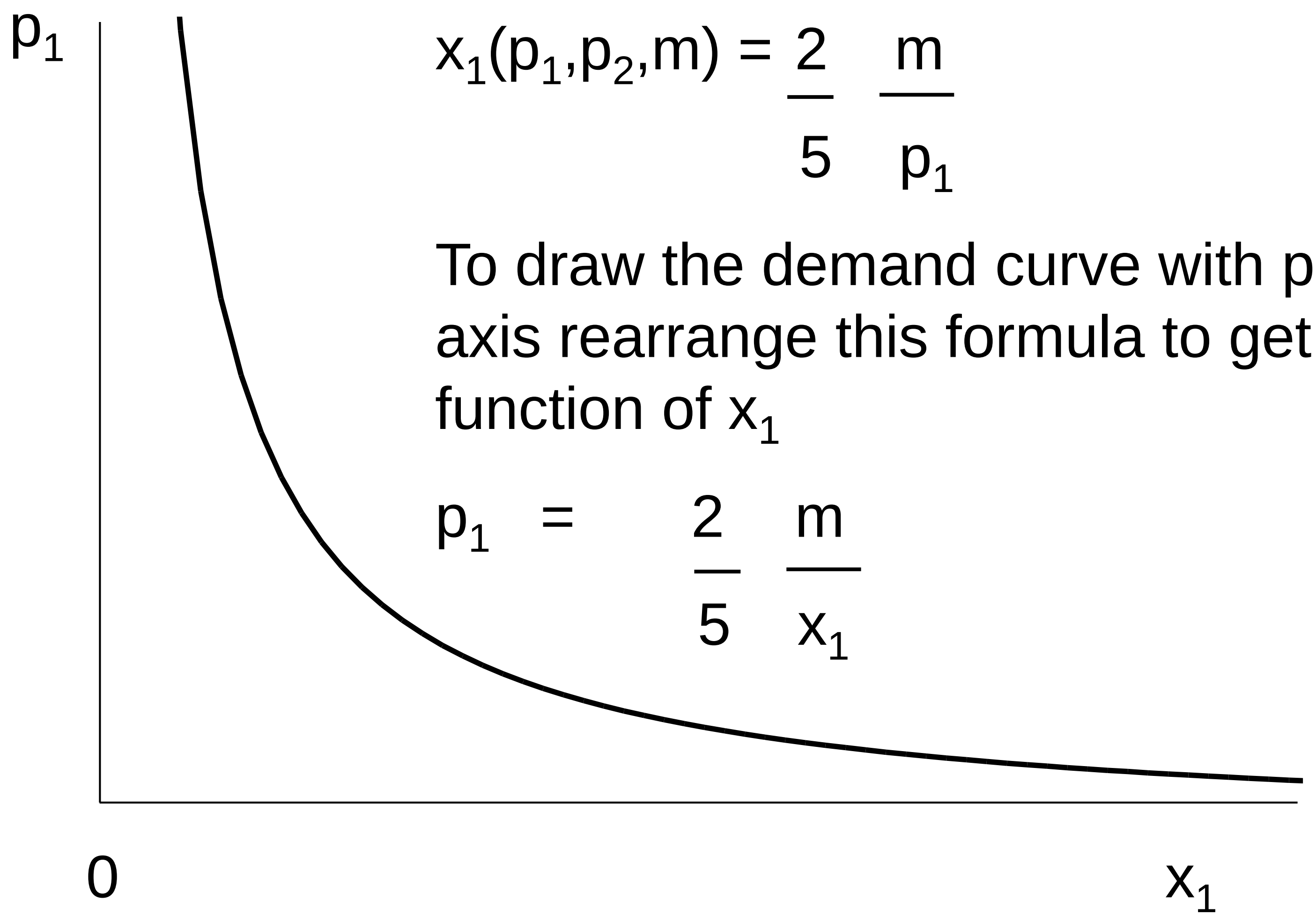
6. Demand curves

Demand

$$x_1(p_1,p_2,m) = \frac{2}{5}\frac{m}{p_1}$$

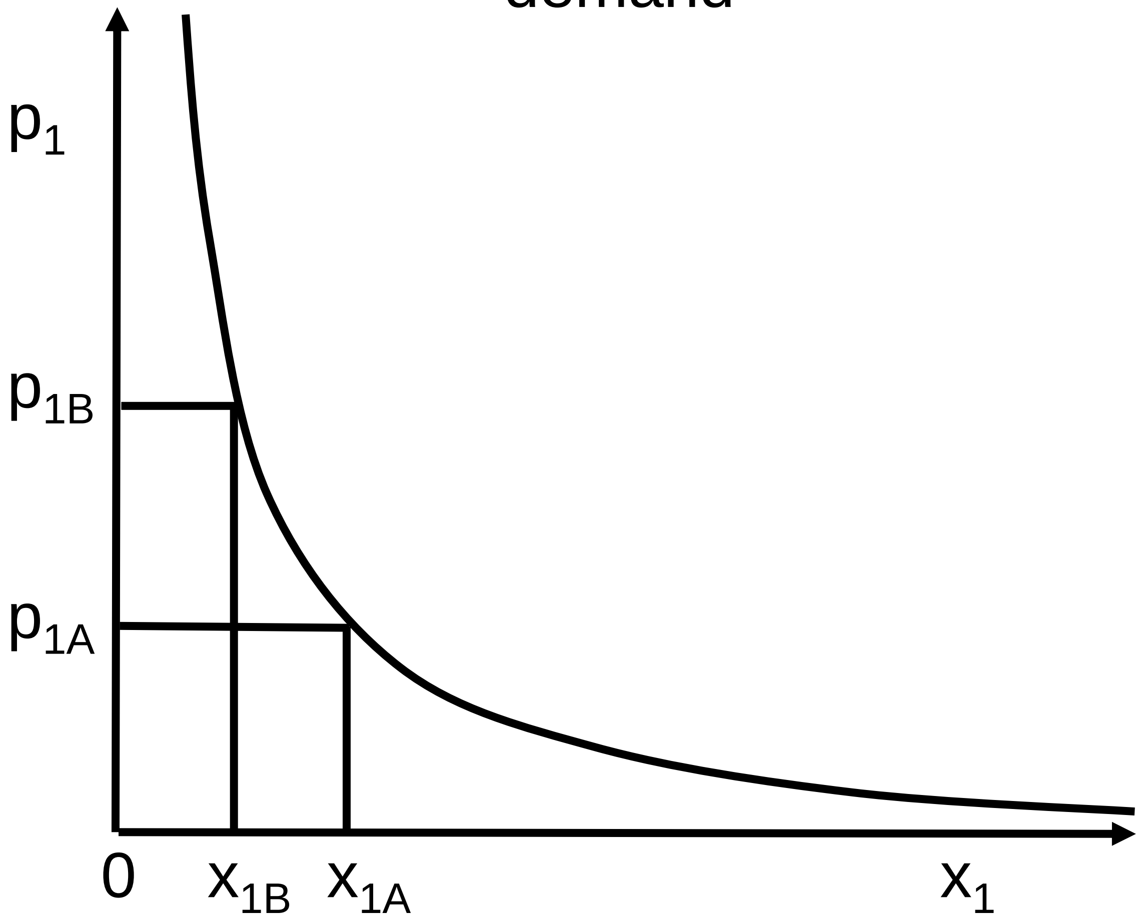
To draw the demand curve with p_1 on the vertical axis rearrange this formula to get p_1 as a function of x_1

$$p_1 = \frac{2}{5}\frac{m}{x_1}$$



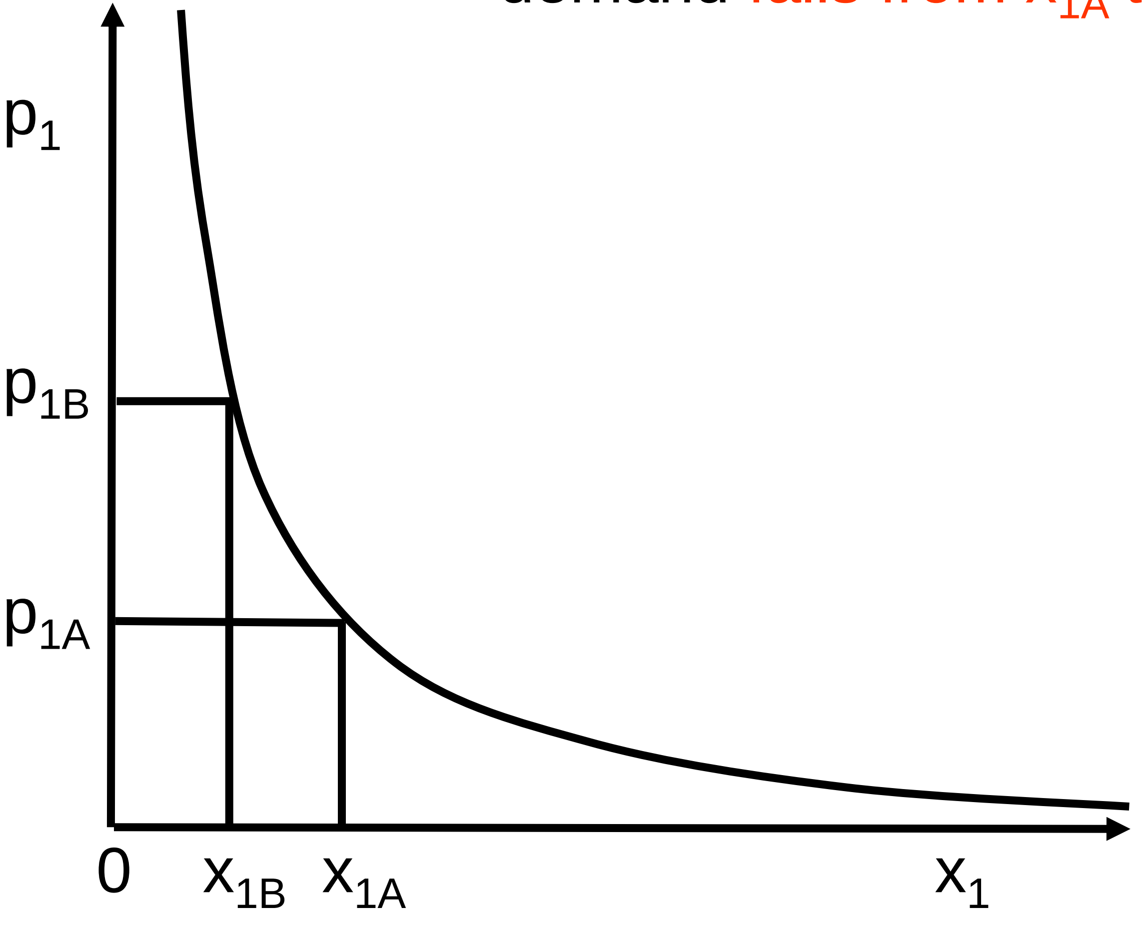
Demand for good 1

If p_1 increases from p_{1A} to p_{1B} there is a movement the demand curve, demand



Demand for good 1

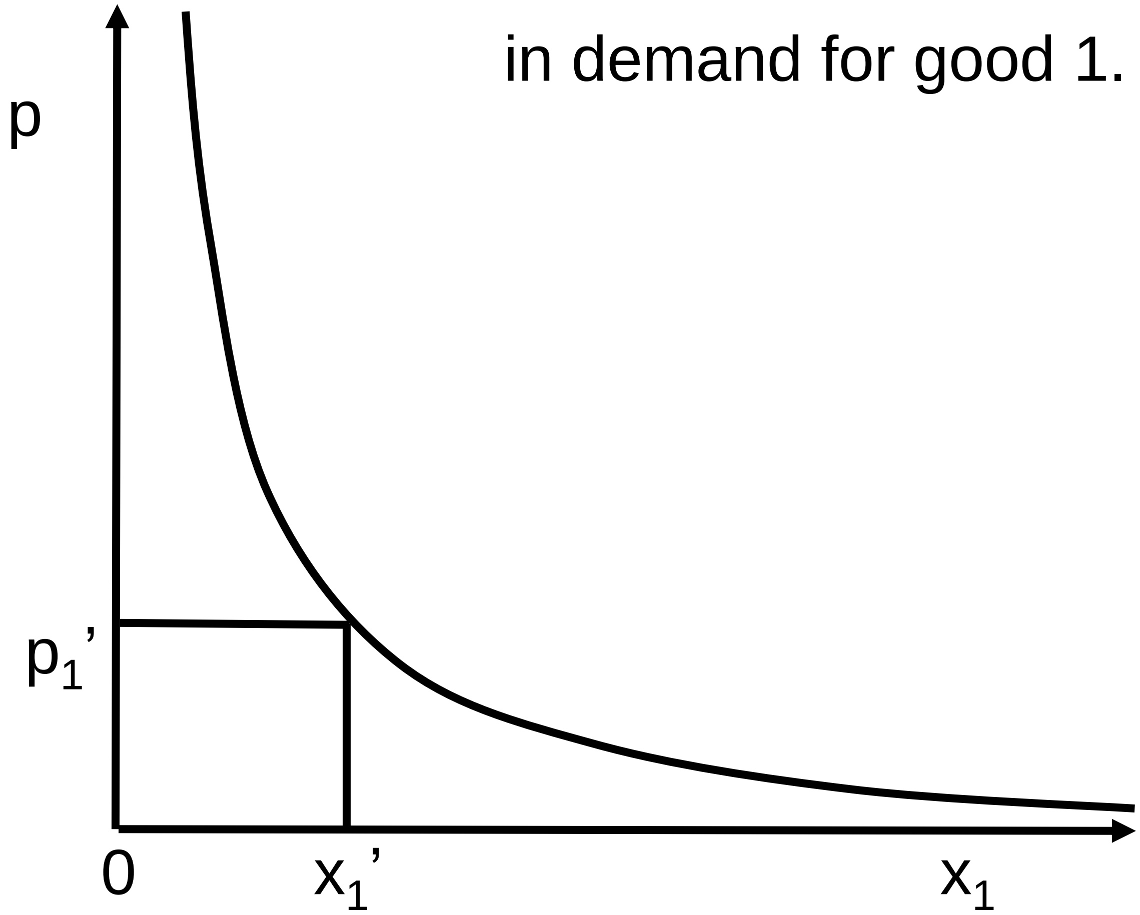
If p_1 increases from p_{1A} to p_{1B} there is a movement on the demand curve, demand falls from x_{1A} to x_{1B}.



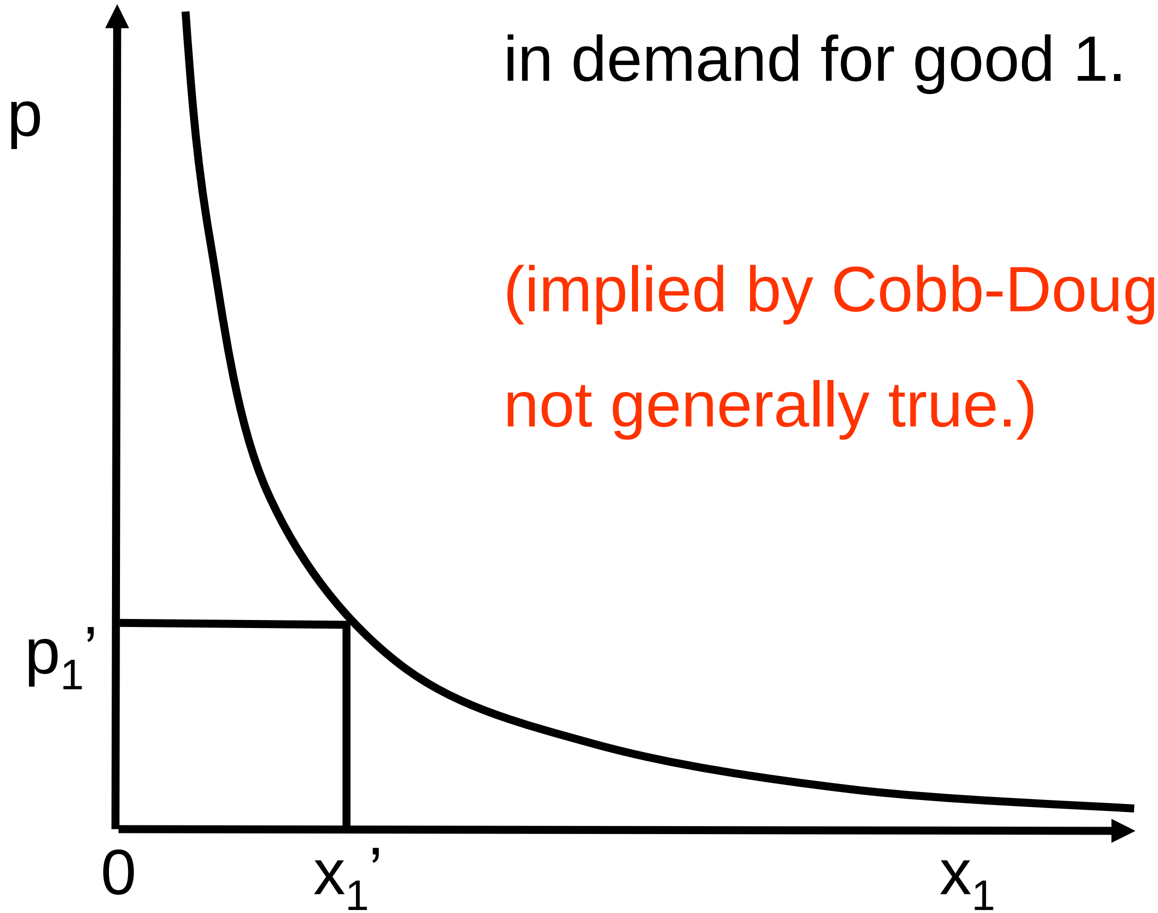
Demand for good 1

If p_2 changes there is

in demand for good 1.



Demand for good 1



If p_2 changes there is no change in demand for good 1.

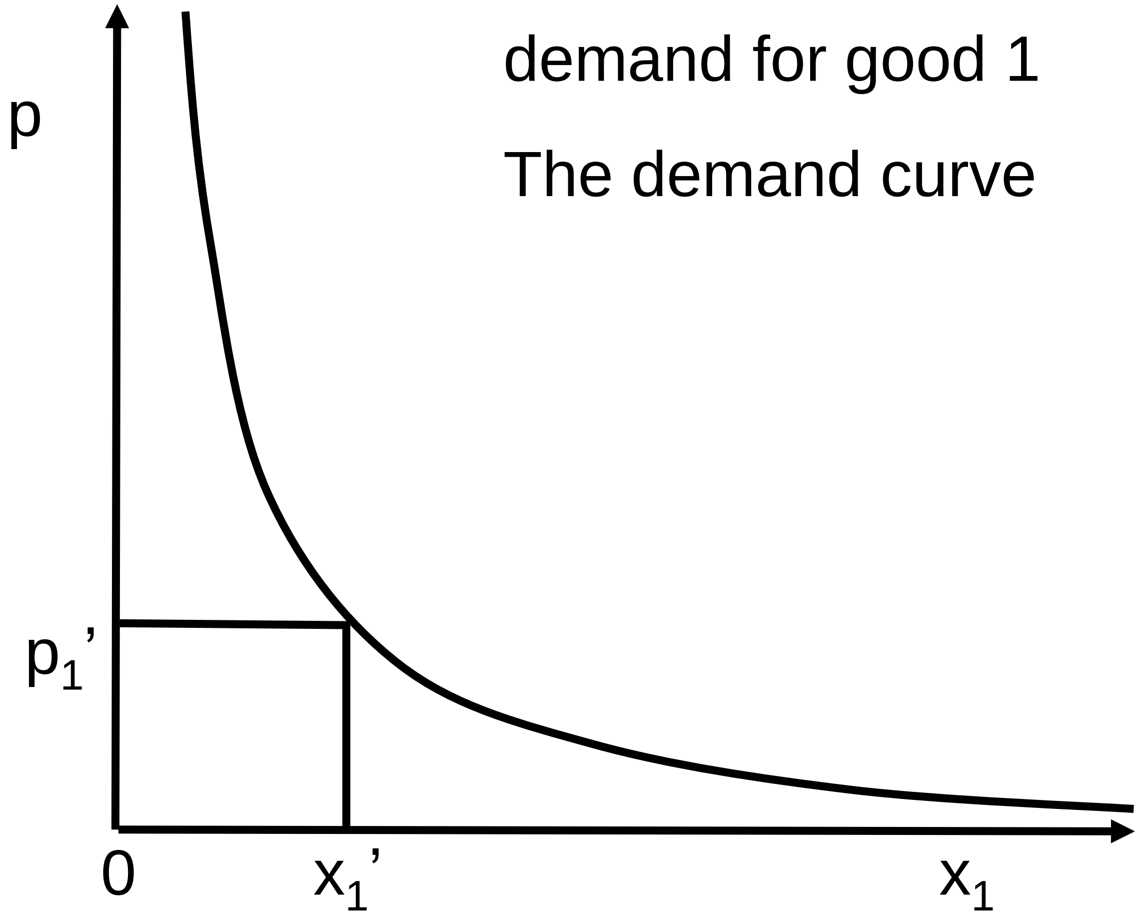
(implied by Cobb-Douglas utility, not generally true.)

Demand for good 1

If income m increases

demand for good 1

The demand curve

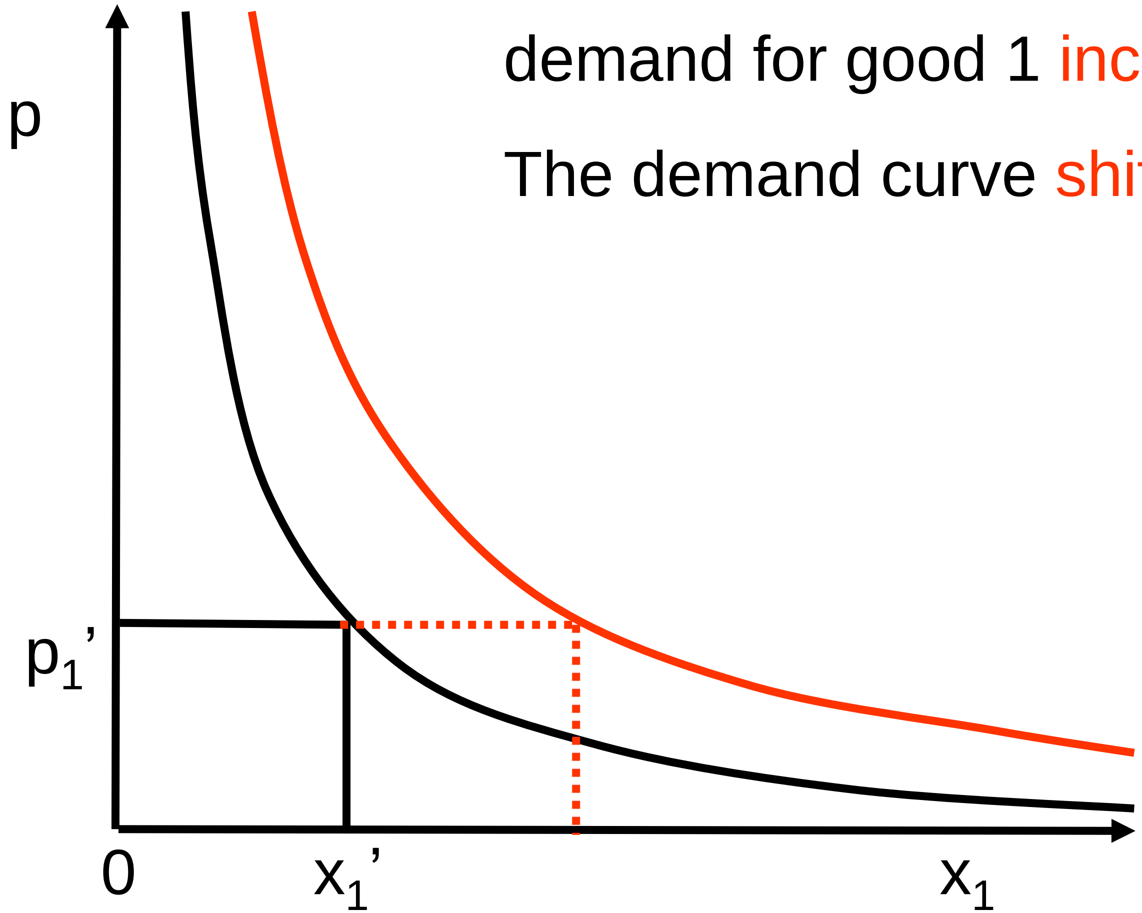


Demand for good 1

If income m increases

demand for good 1 increases.

The demand curve shifts outwards.



Elasticity

7. Elasticity

Measuring the impact of changes in prices & income

Own price elasticity is

$$\frac{\text{\% change in quantity}}{\text{\% change in own price}}$$

Elasticity captures intuition better than

$$\frac{\text{numerical change in quantity}}{\text{numerical change in price}}$$

A price increase from €1 to €2 is large.

A price increase from €10 000 to €10 001 is small.

Elasticity does not depend on units ($ or £, kilos or pounds) because % changes do not depend on units.

Elasticity matters

for every decision on prices, e.g.

for a monopoly or oligopoly deciding on prices

for a government deciding on taxes.

Own price elasticity of demand

either

$$= \frac{\frac{\Delta x_1}{x_1}}{\frac{\Delta p_1}{p_1}} = \frac{\Delta x_1}{\Delta p_1}\frac{p_1}{x_1} \approx \frac{\partial x_1}{\partial p_1}\frac{p_1}{x_1}$$

(negative, Snyder & Nicholson, lectures)

or

$$= -\frac{\frac{\Delta x_1}{x_1}}{\frac{\Delta p_1}{p_1}} = -\frac{\Delta x_1}{\Delta p_1}\frac{p_1}{x_1} \approx -\frac{\partial x_1}{\partial p_1}\frac{p_1}{x_1}$$

Some authors makes elasticity positive

Elasticity and demand curves

Which demand curve is more elastic A or B?

Elasticity and demand curves

Which demand curve is more elastic A or B?

Uncompensated demand for good 1 is

$$x_1(p_1, p_2, m) = \frac{2m}{5p_1}$$

(from Cobb Douglas utility)

Find

own price elasticity $\dfrac{\partial x_1}{\partial p_1} \dfrac{p_1}{x_1}$

Uncompensated demand for good 1 is

$$x_1(p_1, p_2, m) = \frac{2m}{5p_1}$$

(from Cobb Douglas utility)

Find

$$\text{own price elasticity } \frac{\partial x_1}{\partial p_1}\frac{p_1}{x_1} = -\frac{2m}{5p_1^2}\frac{5p_1}{2m}p_1 = -1$$

Uncompensated demand for good 1 is

$$x_1(p_1, p_2, m) = \frac{2m}{5p_1}$$

(from Cobb Douglas utility)

Find

cross price elasticity $\dfrac{\partial x_1}{\partial p_2}\dfrac{p_2}{x_1}$

Uncompensated demand for good 1 is

$$x_1(p_1, p_2, m) = \frac{2m}{5p_1}$$

(from Cobb Douglas utility)

Find

cross price elasticity $\dfrac{\partial x_1}{\partial p_2}\dfrac{p_2}{x_1} = 0\dfrac{5p_1}{2m}p_2 = 0$

Uncompensated demand for good 1 is

$$x_1(p_1, p_2, m) = \frac{2m}{5p_1}$$

(from Cobb Douglas utility)

Find

income elasticity $\quad \dfrac{\partial x_1}{\partial m}\dfrac{m}{x_1}$

Uncompensated demand for good 1 is

$$x_1(p_1, p_2, m) = \frac{2m}{5p_1}$$

Find

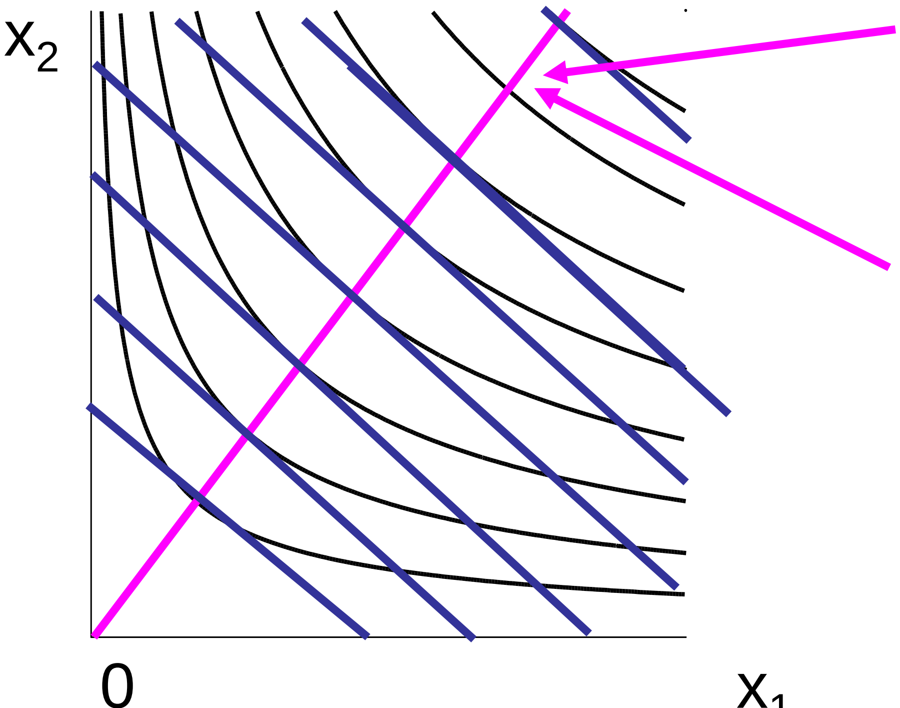
income elasticity $$\frac{\partial x_1}{\partial m}\frac{m}{x_1} = \frac{2}{5p_1}\frac{5p_1}{2m}m = 1$$

With Cobb-Douglas utility $u(x_1,x_2) = x_1^{2/5}x_2^{3/5}$

$$x_1(p_1,p_2,m) = \frac{2}{5}\frac{m}{p_1} \qquad x_2(p_1,p_2,m) = \frac{3}{5}\frac{m}{p_2}$$

MRS = price ratio implies

$$x_2 = \frac{3p_1}{2p_2}x_1$$



income consumption curve

Normal & inferior goods

8. Normal and inferior goods

A good is normal if consumption  when income increases.

A good is inferior if consumption  when income increases.

income elasticity $\dfrac{m}{x_1}\dfrac{\partial x_1}{\partial m} \approx \dfrac{m}{x_1}\dfrac{\Delta x_1}{\Delta m}$

positive if x_1 is a

negative if x_1 is an 

Normal and inferior goods

A good is normal if consumption increases when income increases.

A good is inferior if consumption  when income increases.

income elasticity $\dfrac{m}{x_1}\dfrac{\partial x_1}{\partial m} \approx \dfrac{m}{x_1}\dfrac{\Delta x_1}{\Delta m}$

positive if x_1 is a 

negative if x_1 is an

Normal and inferior goods

A good is normal if consumption increases when income increases.

A good is inferior if consumption decreases when income increases.

income elasticity $\dfrac{m}{x_1}\dfrac{\partial x_1}{\partial m} \approx \dfrac{m}{x_1}\dfrac{\Delta x_1}{\Delta m}$

positive if x_1 is a

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positive if x_1 is a normal good

negative if x_1 is an

Normal and inferior goods

A good is normal if consumption increases when income increases.

A good is inferior if consumption decreases when income increases.

income elasticity $\dfrac{m}{x_1}\dfrac{\partial x_1}{\partial m} \approx \dfrac{m}{x_1}\dfrac{\Delta x_1}{\Delta m}$

positive if x_1 is a normal good

negative if x_1 is an inferior good

Substitutes & complements

9. Substitutes and complements

If demand for good 1 increases when the price of good 2 increases goods 1 and 2 are substitutes.

If demand for good 1 decreases when the price of good 2 increases goods 1 and 2 are complements.

If x_1 and x_2 are substitutes

1. Demand for x_1 increases when p_2 increases.
2. Demand for x_1 decreases when p_2 increases.

If x_1 and x_2 are complements

1. Demand for x_1 increases when p_2 increases.
2.  Demand for x_1 decreases when p_2 increases.

Substitutes and Complements

If demand for good 1 increases when the price of good 2 increases goods 1 and 2 are substitutes.

If demand for good 1 decreases when the price of good 2 increases goods 1 and 2 are complements.

cross price elasticitity $\dfrac{p_2}{x_1}\dfrac{\partial x_1}{\partial p_2} \approx \dfrac{p_2}{x_1}\dfrac{\Delta x_1}{\Delta p_2}$

positive if x_1 and x_2 are

negative if x_1 and x_2 are

Substitutes and Complements

If demand for good 1 increases when the price of good 2 increases goods 1 and 2 are substitutes.

If demand for good 1 decreases when the price of good 2 increases goods 1 and 2 are complements.

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positive if x_1 and x_2 are substitutes

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Substitutes and Complements

If demand for good 1 increases when the price of good 2 increases goods 1 and 2 are substitutes.

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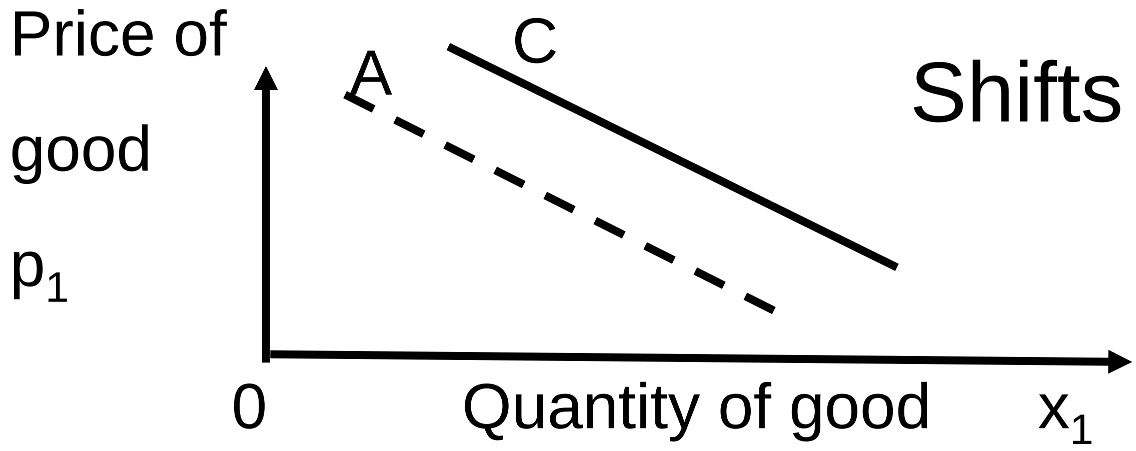
cross price elasticitity $\dfrac{p_2}{x_1}\dfrac{\partial x_1}{\partial p_2} \approx \dfrac{p_2}{x_1}\dfrac{\Delta x_1}{\Delta p_2}$

positive if x_1 and x_2 are substitutes

negative if x_1 and x_2 are complements

Shifts in demand curves



Shift A to C.

This is an increase in demand.

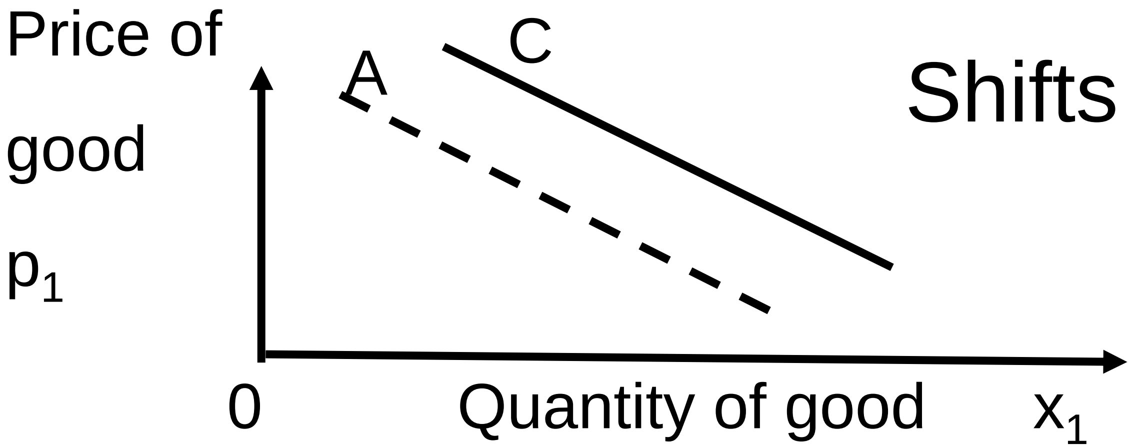
Causes? Increase or decrease in price of a complement?

Increase or decrease in price of a substitute?

Increase or decrease in income for a normal good.

Increase or decrease in income for an inferior good.

Shifts in demand curves



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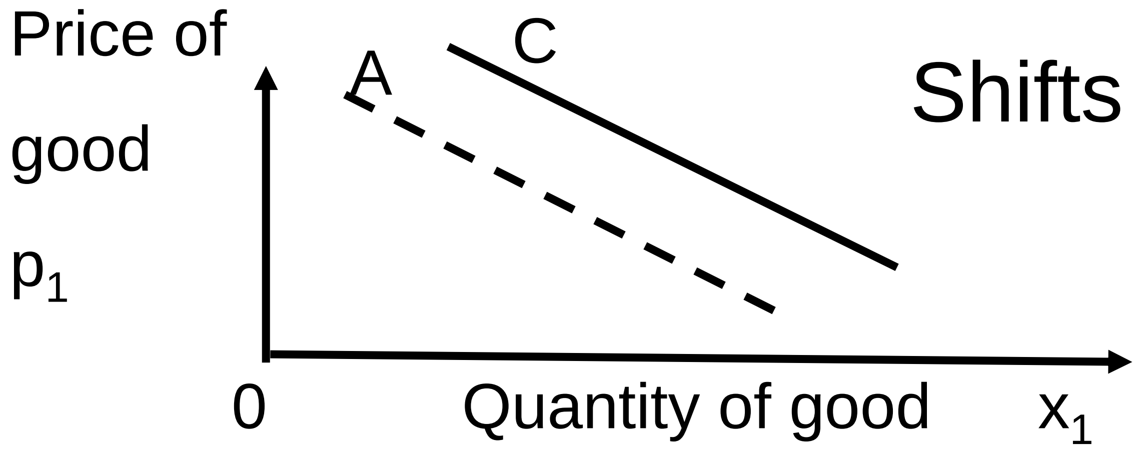
Shifts in demand curves



Shift A to C.

This is an increase in demand.

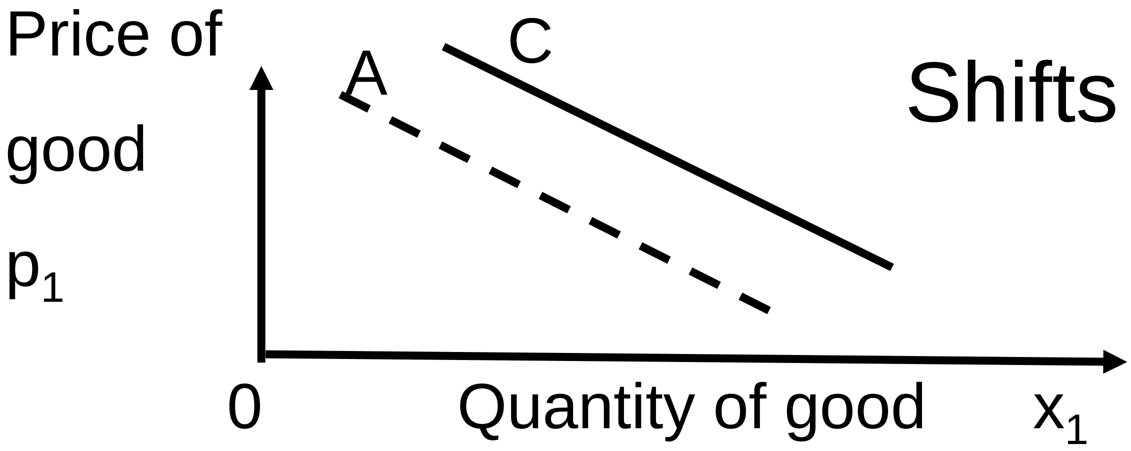
Causes? Increase or decrease in price of a complement?

Increase or decrease in price of a substitute?

Increase or decrease in income for a normal good.

Increase or decrease in income for an inferior good.

Shifts in demand curves



Shift A to C.

This is an increase in demand.

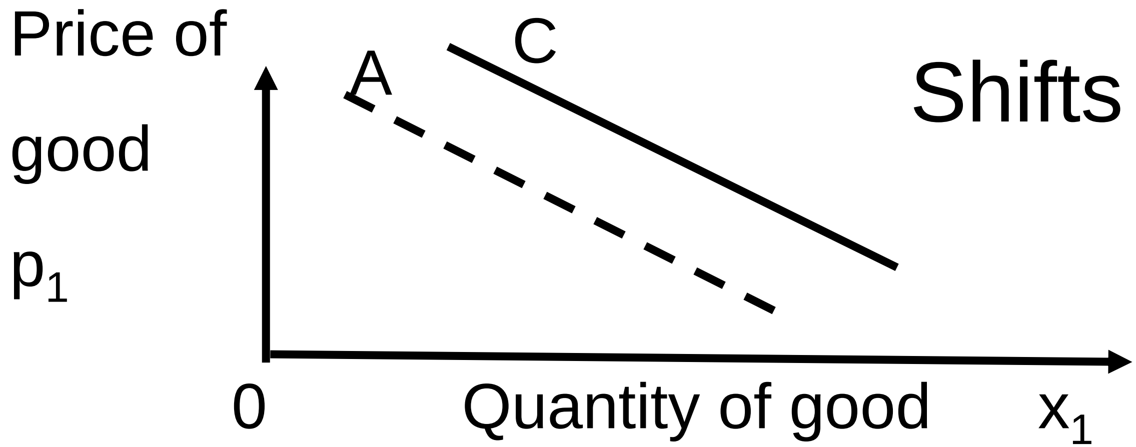
Causes? Increase or decrease in price of a complement?

Increase or decrease in price of a substitute?

Increase or decrease in income for a normal good.

Increase or decrease in income for an inferior good.

Shifts in demand curves



Shift A to C.

This is an increase in demand.

Causes? Increase or decrease in price of a complement?

Increase or decrease in price of a substitute?

Increase or decrease in income for a normal good.

Increase or decrease in income for an inferior good.

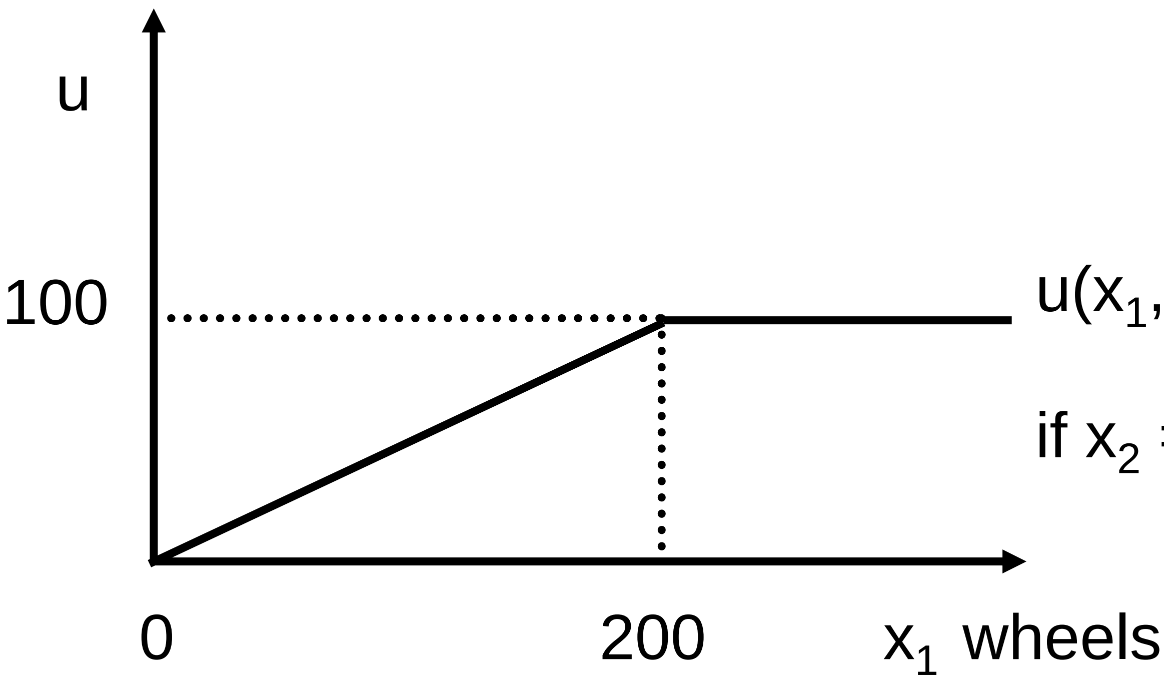
Finding uncompensated demand with perfect complements utility

10. Finding uncompensated demand with perfect complements utility

In general $u(x_1,x_2) = \min(ax_1,bx_2)$

here $u(x_1,x_2) = \min(½\, x_1, x_2)$

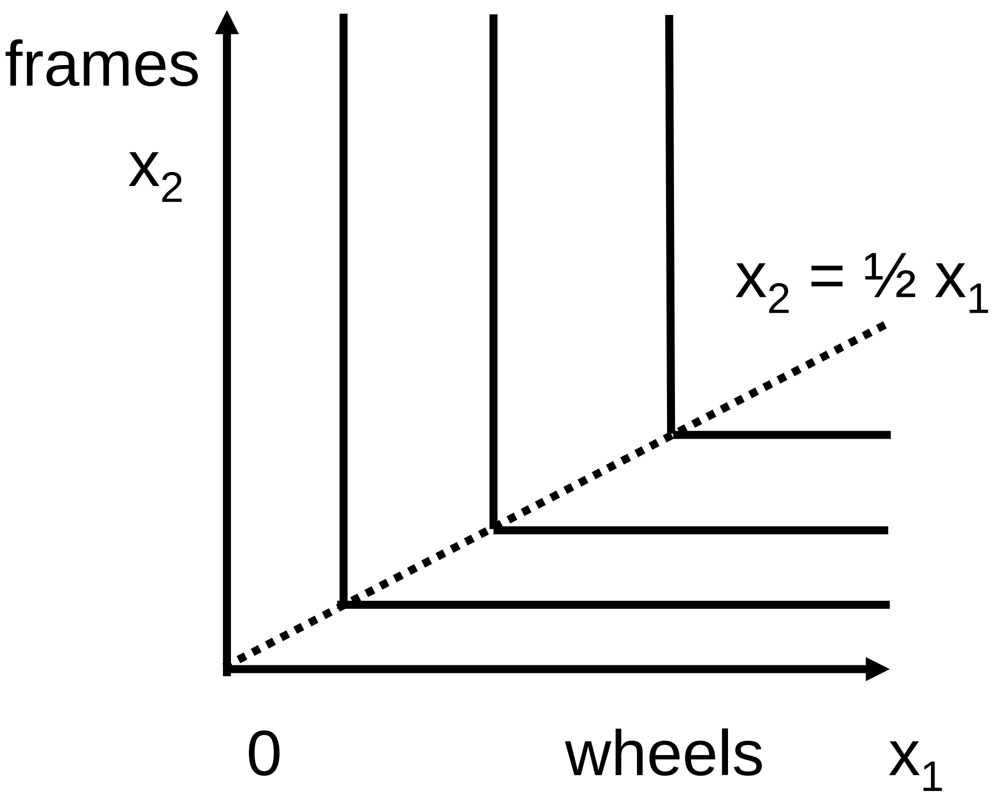
x_1 bike wheels, x_2 bicycle frames



$u(x_1,x_2) = \min(½\, x_1,100)$

if $x_2 = 100$.

Perfect complements utility: indifference curves



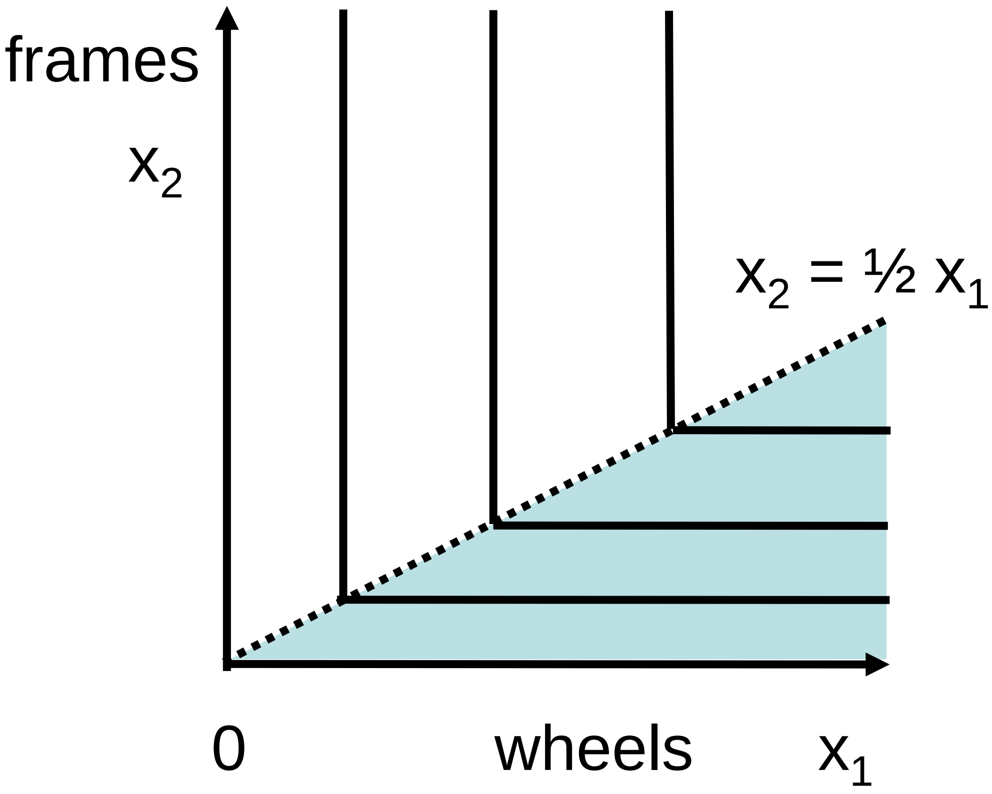
$u(x_1,x_2) = \min(\,½\, x_1, x_2)$

x_1 bicycle wheels,
x_2 bicycle frames

if $x_2 < ½\, x_1$ increasing x_1 does not change utility

if $x_2 > ½\, x_1$ increasing x_1 increases utility.

Perfect complements utility: indifference curves



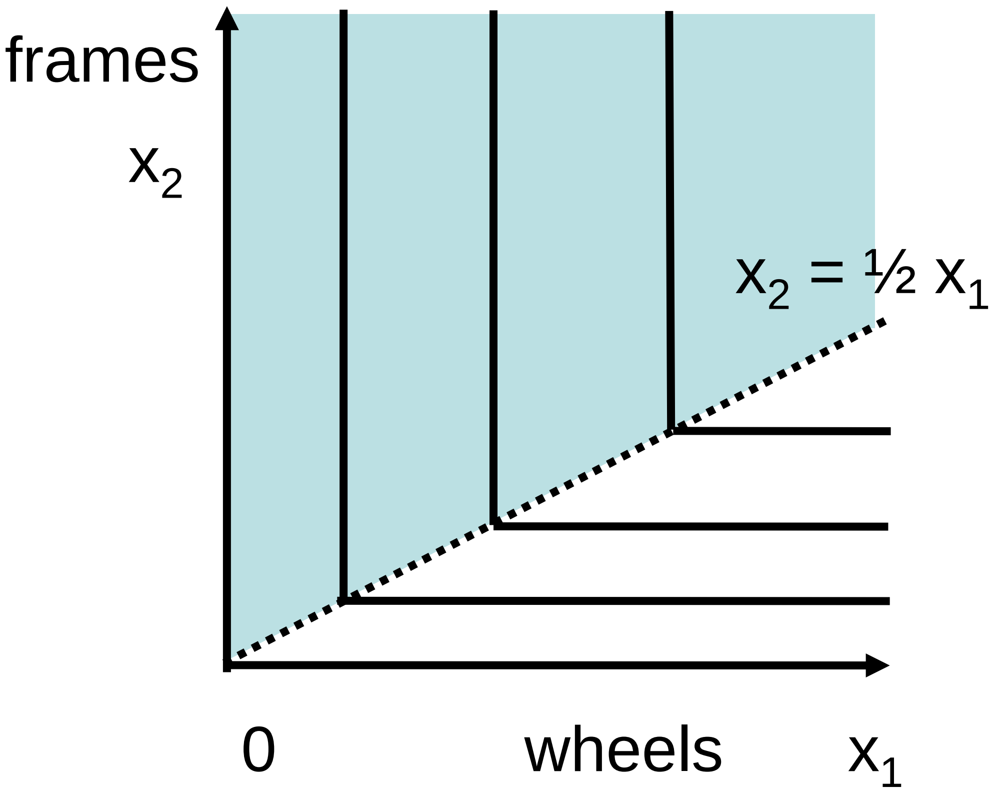
$u(x_1,x_2) = \min(½ x_1, x_2)$

x_1 bicycle wheels,
x_2 bicycle frames

if $x_2 < ½ x_1$ increasing x_1 does not change utility

if $x_2 > ½ x_1$ increasing x_1 increases utility.

Perfect complements utility: indifference curves



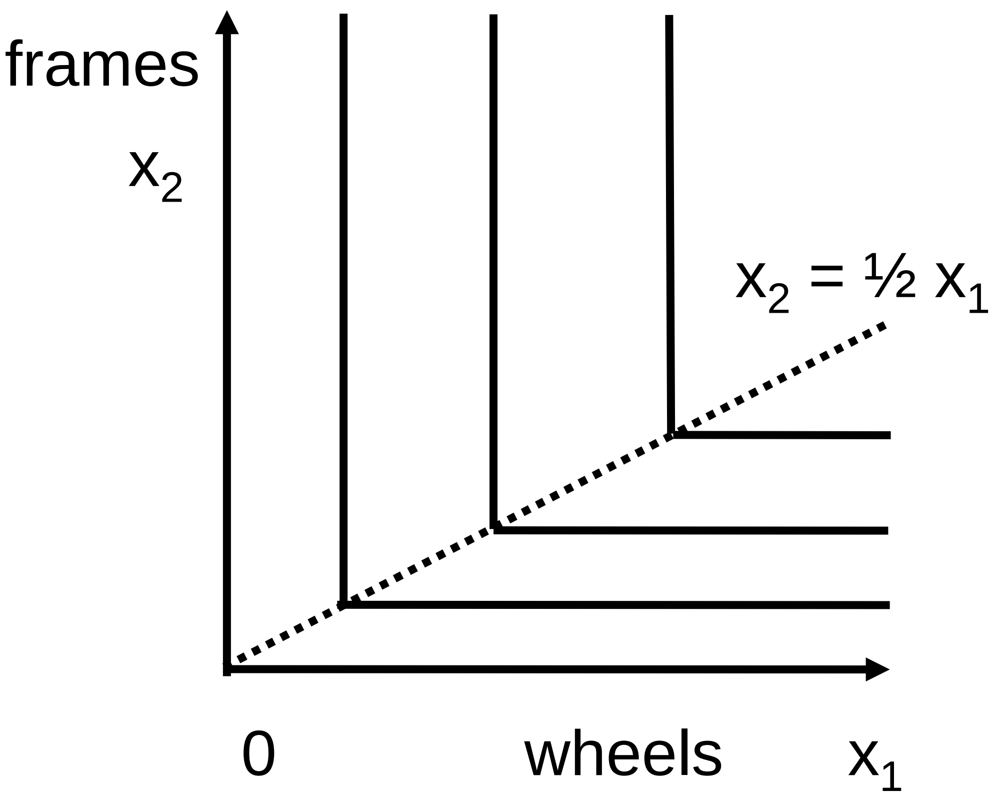
$u(x_1,x_2) = \min(\tfrac{1}{2} x_1, x_2)$

x_1 bicycle wheels,
x_2 bicycle frames

if $x_2 < ½\ x_1$ increasing x_1 does not change utility

if $x_2 > ½\ x_1$ increasing x_1 increases utility.

Perfect complements utility: indifference curves

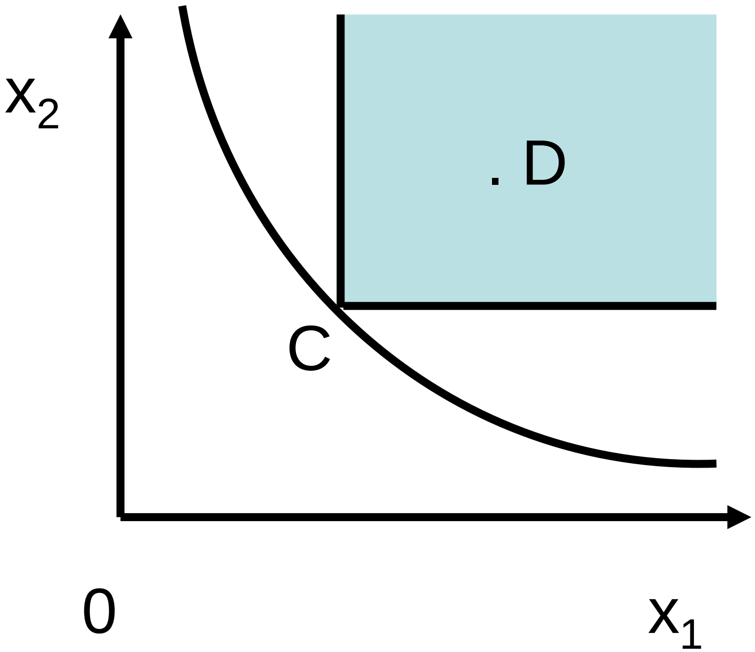


$u(x_1,x_2) = \min(½ x_1, x_2)$

x_1 bicycle wheels,
x_2 bicycle frames

if $x_2 = ½ x_1$ it is necessary to increase both x_1 and x_2 to increase utility

Nonsatiation in the indifference curve diagram with **differentiable utility**

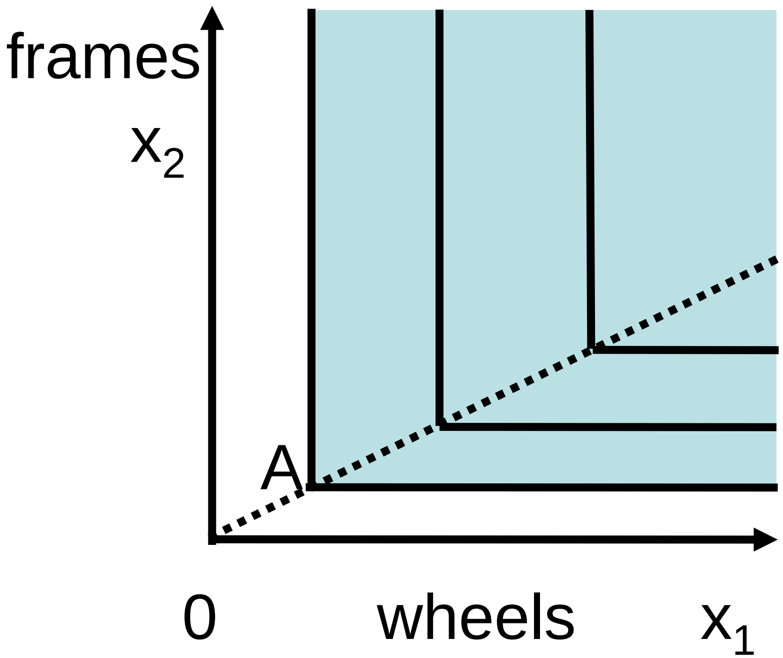


Nonsatiation means that any point such as D inside or on the boundary of the shaded area is preferred to C.

Here starting from C increasing x_1 and/or increasing x_2 increases utility.

Check for this by seeing if the partial derivatives of utility function are > 0.

Nonsatiation in the indifference curve diagram with **perfect complements utility**



Here starting from A increasing x_1 and x_2 increases utility.

Increasing only x_1 or only x_2 does not increase utility.

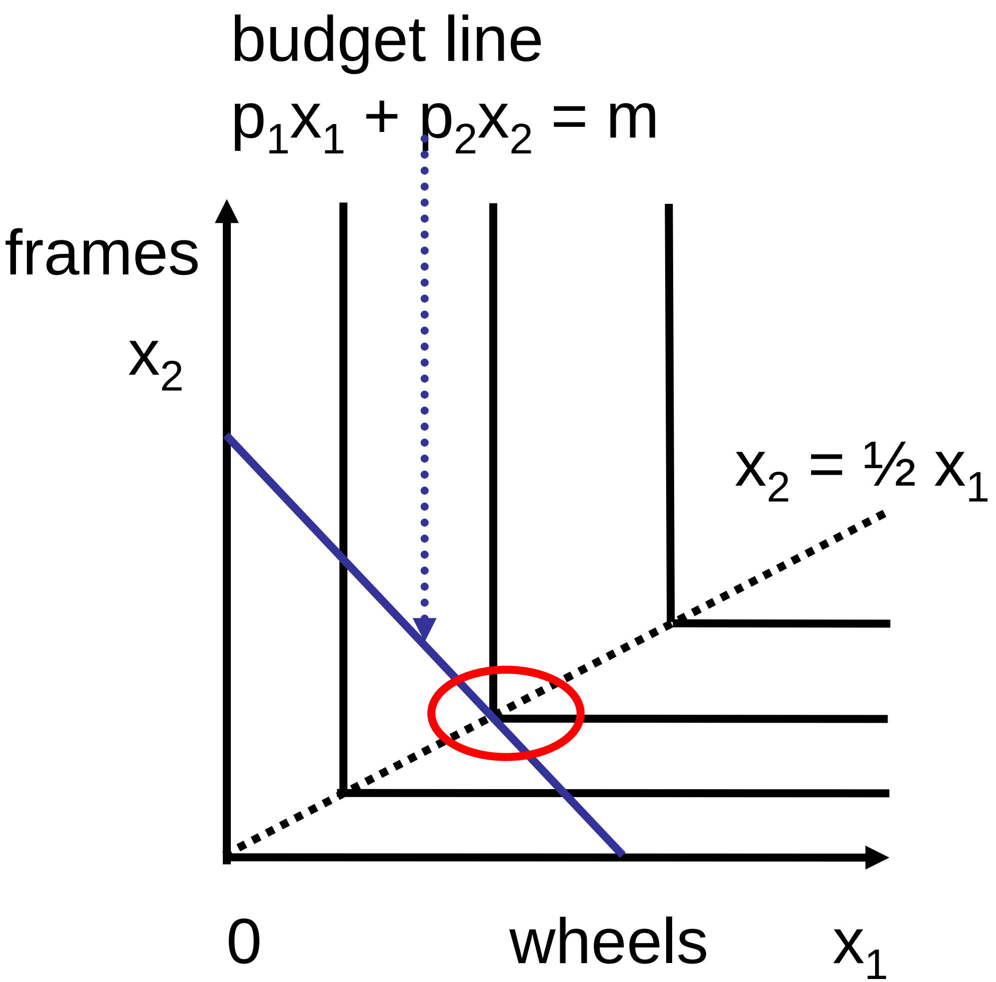
(Think about frames & wheels.)

The function $u(x_1,x_2) = \min(\frac{1}{2} x_1, x_2)$

does not have partial derivatives when $\frac{1}{2} x_1 = x_2$

does not have MRS. Can't use calculus.

Perfect complements: utility maximization



$u(x_1,x_2) = \min(½ x_1,x_2)$

utility maximization

implies that (x_1,x_2)

lies at the kink of the indifference curves so

$x_2 = ½ x_1$

and satisfies the budget constraint so

$p_1x_1 + p_2x_2 = m.$

Perfect complements: utility maximization

$x_2 = \frac{1}{2} x_1$

$p_1x_1 + p_2x_2 = m.$

Solving simultaneously for x_1 and x_2 gives

$$x_1 = \frac{2m}{(2p_1 + p_2)} \qquad x_2 = \frac{m}{(2p_1 + p_2)}$$

Common mistake

x_1 wheels, x_2 frames,

wheels *frames*

2 wheels for each frame

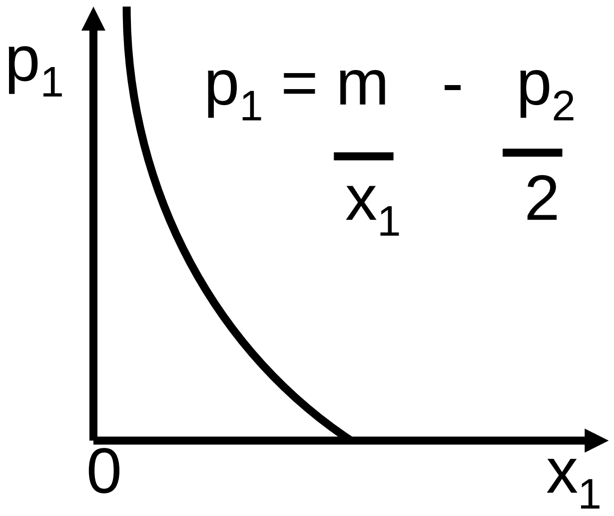
Easy to think that utility should be $u(x_1,x_2) = \min(2x_1, x_2)$

But this implies that $x_2 = 2x_1$,

number of frames = 2 (number of wheels)

Utility is $u(x_1,x_2) = \min(½\, x_1, x_2)$

Demand curves and changes in prices and income with perfect complements utility



demand curve diagram,

price on vertical axis

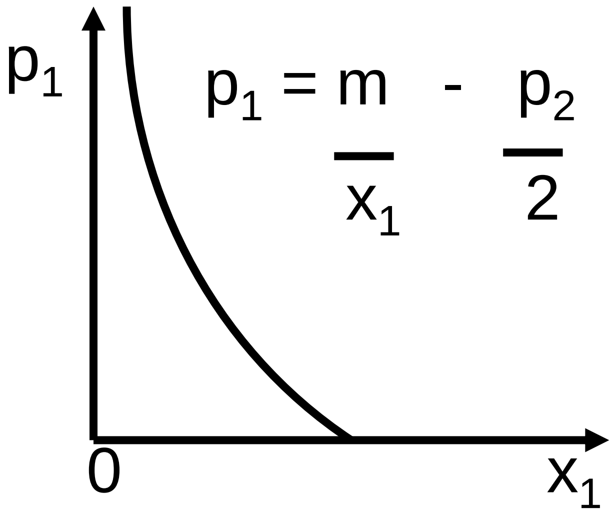
quantity on horizontal axis

Increase in p_1 results in

Increase in p_2 results in

Increase in m results in

Demand curves and changes in prices and income with perfect complements utility



demand curve diagram,

price on vertical axis

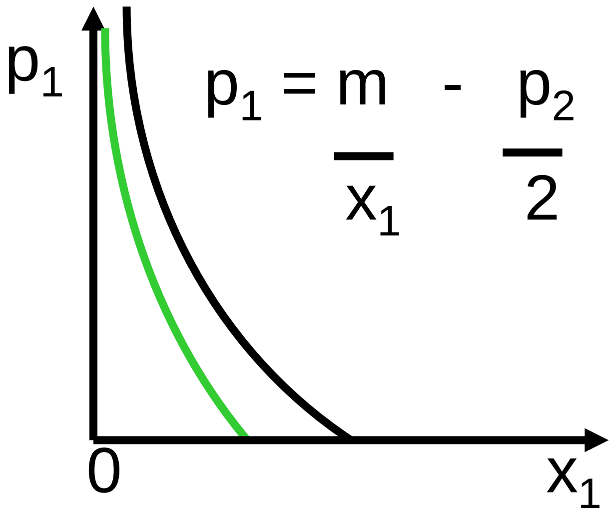
quantity on horizontal axis

Increase in p_1 results in movement along demand curve.

Increase in p_2 results in

Increase in m results in

Demand curves and changes in prices and income with perfect complements utility



demand curve diagram,

price on vertical axis

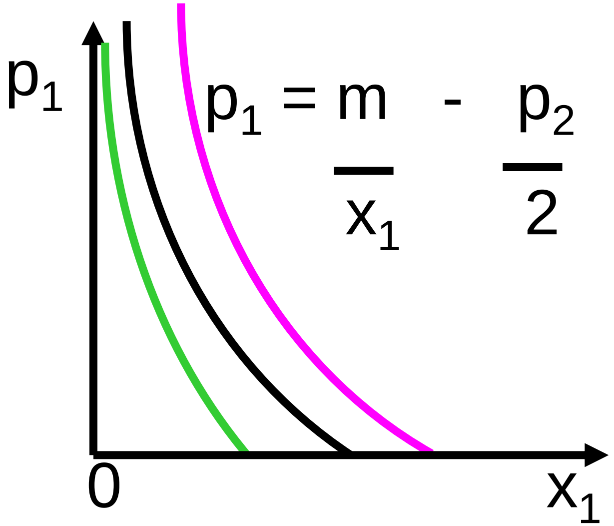
quantity on horizontal axis

Increase in p_1 results in movement along demand curve.

Increase in p_2 results in shift down in demand curve.

Increase in m results in

Demand curves and changes in prices and income with perfect complements utility



demand curve diagram,

price on vertical axis

quantity on horizontal axis

Increase in p_1 results in movement along demand curve.

Increase in p_2 results in shift down in demand curve.

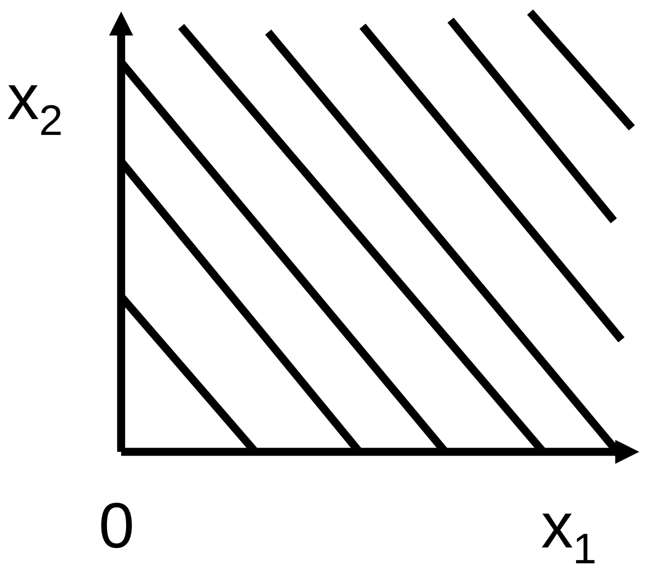
Increase in m results in shift up in demand curve.

Finding uncompensated demand with perfect substitutes utility: corner solutions again

Perfect substitutes utility

In general $u(x_1,x_2) = ax_1 + bx_2$

$u(x_1,x_2) = 3x_1 + 2x_2$.



indifference curves

$u = 3x_1 + 2x_2$

gradient $-3/2$

11. Finding uncompensated demand with perfect substitutes utility

Step 1: What problem are you solving?

The problem is maximizing utility $u(x_1,x_2) = 3x_1 + 2x_2$

subject to non-negativity constraints $x_1 \geq 0 \quad x_2 \geq 0$

and the budget constraint $p_1x_1 + p_2x_2 \leq m$.

Step 2: What is the solution a function of?

Demand is a function of prices and income so is

$x_1(p_1,p_2,m) \qquad x_2(p_1,p_2,m)$

Finding uncompensated demand with perfect substitutes utility

Step 3: Check for nonsatiation and convexity

$\frac{\partial u}{\partial x_1} = 3 > 0, \ \frac{\partial u}{\partial x_2} = 2 > 0$ so nonsatiation is satisfied.

Getting x_2 as a function of u and x_1 gives $x_2 = (u - 3x_1)/2$ so

$$\frac{\partial x_2}{\partial x_1} = -3/2 \qquad \frac{\partial^2 x_2}{\partial x^2_1} = 0$$

convexity is satisfied.

Finding uncompensated demand with perfect substitutes utility

Step 4: Use the tangency and budget line conditions

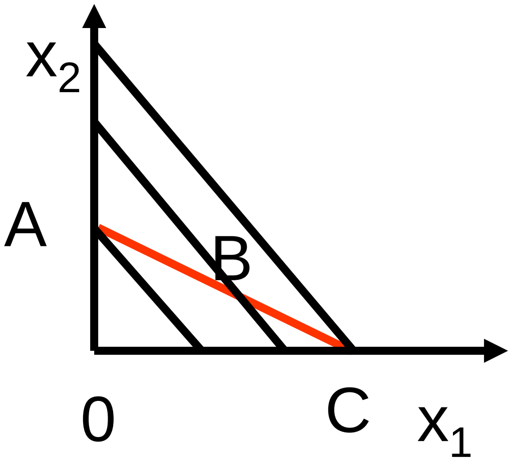
Because convexity and nonsatiation are satisfied any point with

$p_1x_1 + p_2x_2 = m$ so is on the budget line

and MRS = $\dfrac{p_1}{p_2}$

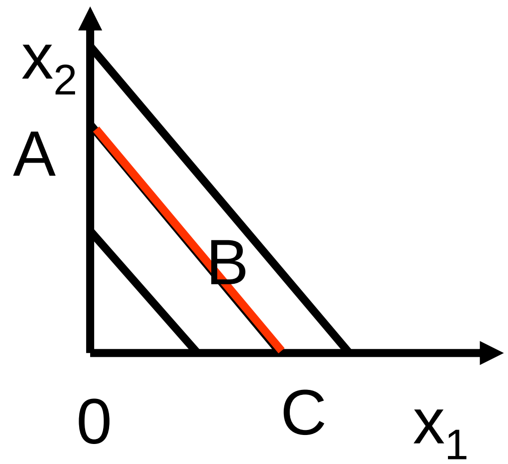
$$MRS = \frac{\dfrac{\partial u}{\partial x_1}}{\dfrac{\partial u}{\partial x_2}} = \frac{3}{2}$$

> Problem
>
> What if $p_1/p_2 \neq 3/2$?
>
> It is better to use a diagram.



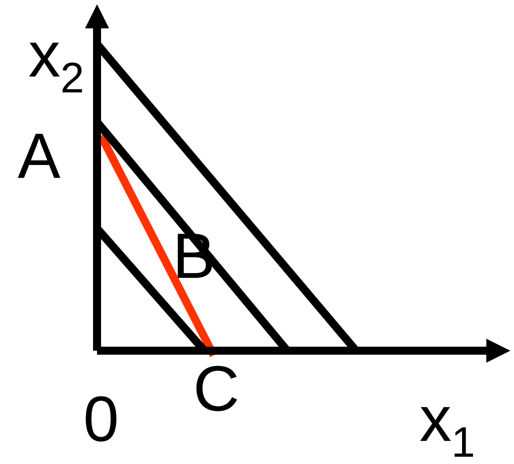
$p_1/p_2 < 3/2$

solution at



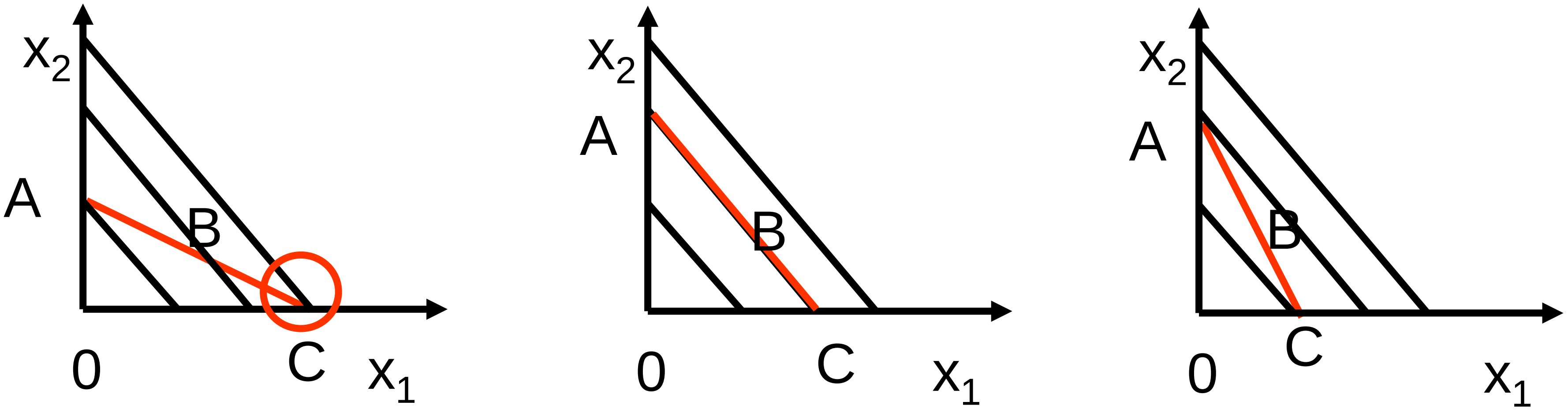
$p_1/p_2 = 3/2$

solution at



$p_1/p_2 > 3/2$

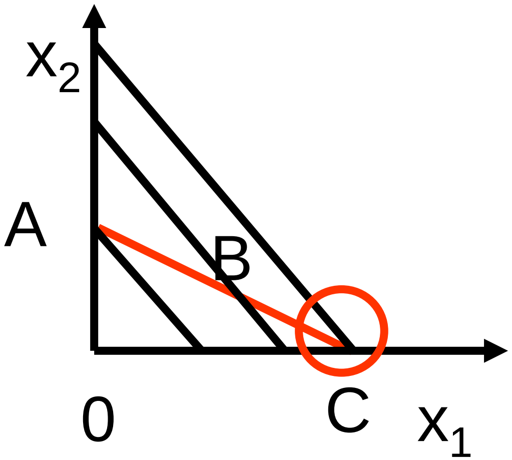
solution at

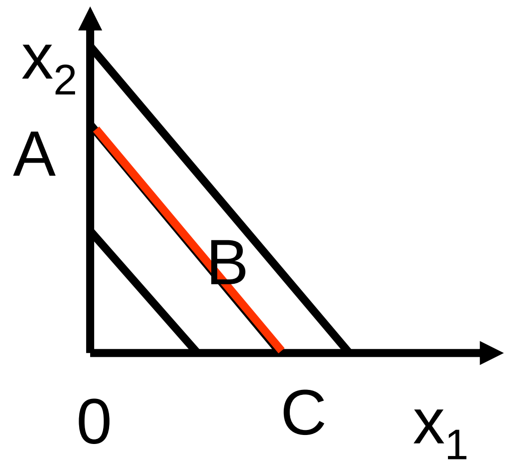


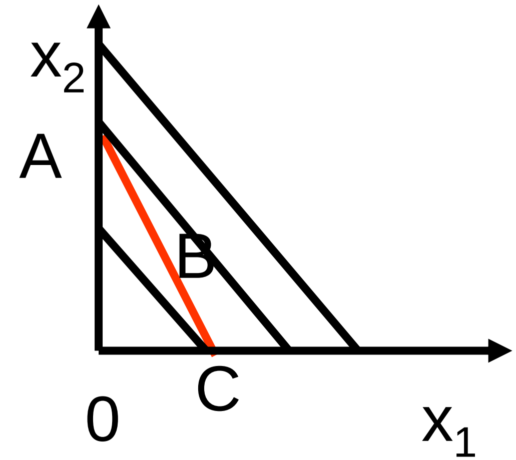
$p_1/p_2 < 3/2$

solution at C

$x_1 = m/p_1,\ x_2 = 0$







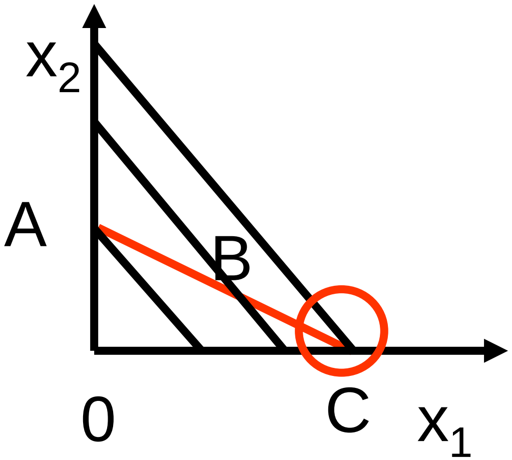
$p_1/p_2 < 3/2$

solution at C

$x_1 = m/p_1, x_2 = 0$

$p_1/p_2 = 3/2$

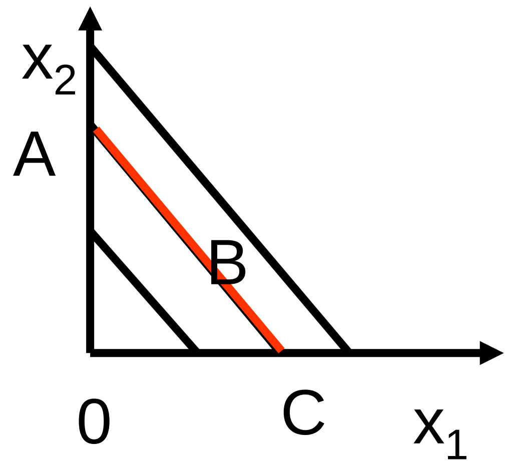
solution at



$p_1/p_2 < 3/2$

solution at C

$x_1 = m/p_1,\ x_2 = 0$

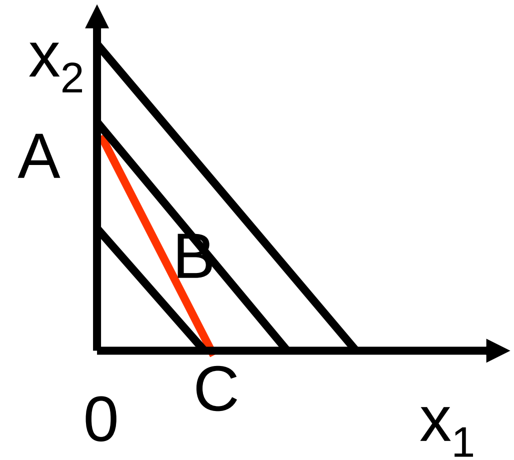


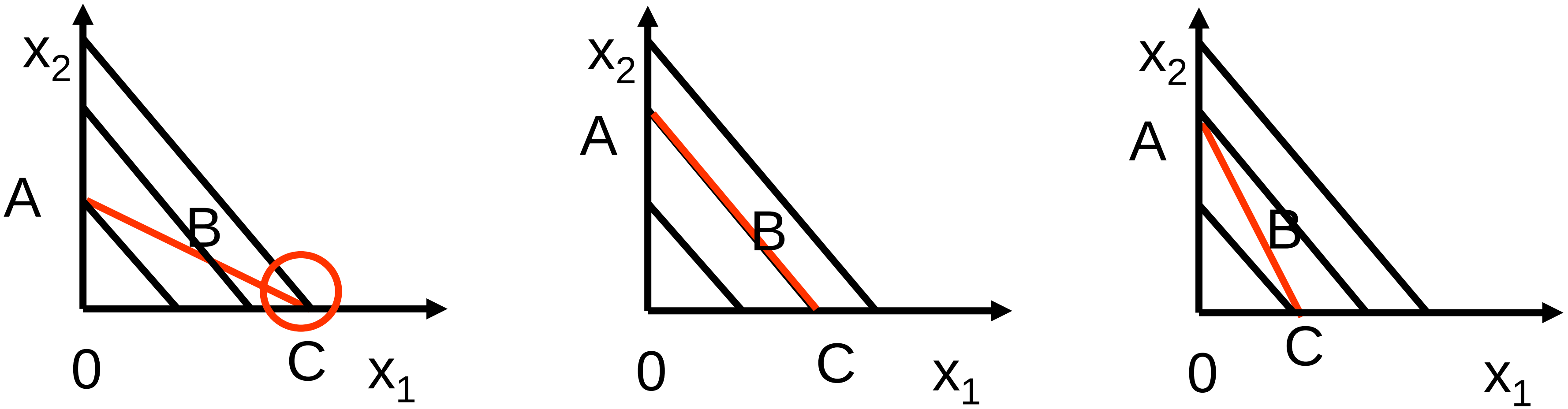
$p_1/p_2 = 3/2$

solution at any x_1 x_2

satisfying $x_1 \geq 0$

$x_2 \geq 0$ and budget constraint





$p_1/p_2 < 3/2$

solution at C

$x_1 = m/p_1$, $x_2 = 0$

$p_1/p_2 = 3/2$

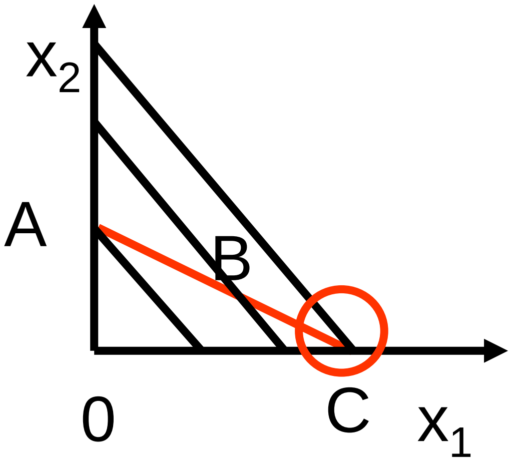
solution at any x_1 x_2

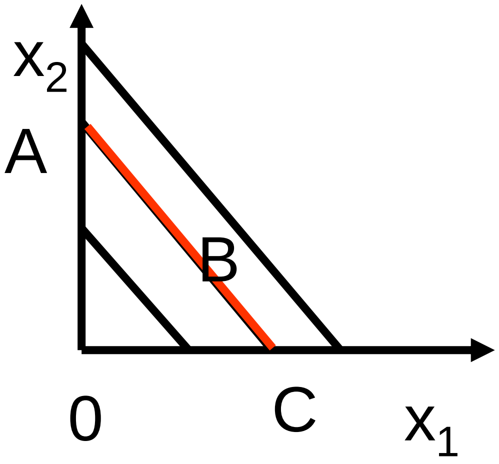
satisfying $x_1 \geq 0$

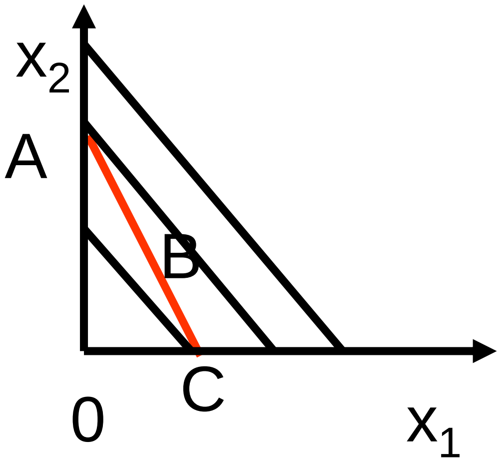
$x_2 \geq 0$ and budget constraint

$p_1/p_2 > 3/2$

solution at







$p_1/p_2 < 3/2$

solution at C

$x_1 = m/p_1$, $x_2 = 0$

$p_1/p_2 = 3/2$

solution at any x_1 x_2

satisfying $x_1 \geq 0$

$x_2 \geq 0$ and budget constraint

$p_1/p_2 > 3/2$

solution at

A $x_1 = 0$

$x_2 = m/p_2$.

What have we achieved?

- Model of consumer demand: given preferences satisfying listed assumptions.
- Show that preferences can be represented by utility functions: mathematically convenient.
- Model shows how demand responds to changes in own price, price of other good, income.
- Model has only two goods, but with more maths can easily be extended to many goods.